# NAVAL POSTGRADUATE SCHOOL

**MONTEREY, CALIFORNIA**

# THESIS

**TRI-LEVEL OPTIMIZATION FOR ANTI-SUBMARINE WARFARE MISSION PLANNING**

by

Adam J. Thomas

September 2008

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| Thesis Advisors: | Gerald G. Brown |
| | R. Kevin Wood |
| Second Reader: | Jeffrey E. Kline |

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| REPORT DOCUMENTATION PAGE | | | *Form Approved OMB No. 0704-0188* |
|---|---|---|---|
| Public reporting burden for this collection of information is estimated to average 1 hour per response, including the time for reviewing instruction, searching existing data sources, gathering and maintaining the data needed, and completing and reviewing the collection of information. Send comments regarding this burden estimate or any other aspect of this collection of information, including suggestions for reducing this burden, to Washington headquarters Services, Directorate for Information Operations and Reports, 1215 Jefferson Davis Highway, Suite 1204, Arlington, VA 22202-4302, and to the Office of Management and Budget, Paperwork Reduction Project (0704-0188) Washington DC 20503. | | | |
| **1. AGENCY USE ONLY *(Leave blank)*** | **2. REPORT DATE** September 2008 | **3. REPORT TYPE AND DATES COVERED** Master's Thesis | |
| **4. TITLE AND SUBTITLE** Tri-Level Optimization for Anti-Submarine Warfare Mission Planning | | | **5. FUNDING NUMBERS** |
| **6. AUTHOR(S)** Adam J. Thomas | | | |
| **7. PERFORMING ORGANIZATION NAME(S) AND ADDRESS(ES)** Naval Postgraduate School Monterey, CA 93943-5000 | | | **8. PERFORMING ORGANIZATION REPORT NUMBER** |
| **9. SPONSORING /MONITORING AGENCY NAME(S) AND ADDRESS(ES)** N/A | | | **10. SPONSORING/MONITORING AGENCY REPORT NUMBER** |
| **11. SUPPLEMENTARY NOTES** The views expressed in this thesis are those of the author and do not reflect the official policy or position of the Department of Defense or the U.S. Government. | | | |
| **12a. DISTRIBUTION / AVAILABILITY STATEMENT** Approved for public release; distribution is unlimited | | | **12b. DISTRIBUTION CODE** |

**13. ABSTRACT (maximum 200 words)**

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| **14. SUBJECT TERMS** Optimization, Mathematical Programming, Anti-Submarine Warfare, Search and Detection, Defender-Attacker Optimization, Tri-Level Optimization, Game Theory | | | **15. NUMBER OF PAGES** 107 |
|---|---|---|---|
| | | | **16. PRICE CODE** |
| **17. SECURITY CLASSIFICATION OF REPORT** Unclassified | **18. SECURITY CLASSIFICATION OF THIS PAGE** Unclassified | **19. SECURITY CLASSIFICATION OF ABSTRACT** Unclassified | **20. LIMITATION OF ABSTRACT** UU |

NSN 7540-01-280-5500

Standard Form 298 (Rev. 2-89)
Prescribed by ANSI Std. 239-18

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**TRI-LEVEL OPTIMIZATION**
**FOR ANTI-SUBMARINE WARFARE MISSION PLANNING**

Adam J. Thomas
Lieutenant, United States Navy
BSE, University of Michigan, 2002

Submitted in partial fulfillment of the
requirements for the degree of

**MASTER OF SCIENCE IN OPERATIONS RESEARCH**

from the

**NAVAL POSTGRADUATE SCHOOL**
**September 2008**

Author: Adam J. Thomas

Approved by: Gerald G. Brown
Distinguished Professor of Operations Research
Thesis Advisor

R. Kevin Wood
Professor of Operations Research
Thesis Advisor

Jeffrey E. Kline
Senior Lecturer in Operations Research
CAPT-USN (Ret)
Second Reader

James N. Eagle
Chairman, Department of Operations Research

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# ABSTRACT

We develop the Game-Theoretic ASW Mission Planner (G-TAMP), an operational-level planning aid for the tasking of anti-submarine warfare (ASW) platforms to protect a high-value unit (HVU) from attack by hostile submarines (SSKs). We first present a defender-attacker optimization model in which the defender tasks platforms to minimize the probability that the enemy can reach the HVU, while the enemy observes and reacts to these visible defenses by routing SSKs to maximize this probability. A defender-attacker/defender (D-A/D) model then extends the first model by adding a final "defender stage" to task potentially "secret" platforms. This model also prescribes the optimal sensor mode for platforms that can use passive sonar (for secrecy) or active sonar (for increased detection ranges), in effect, quantifying the value of secrecy for the defender. Five scenarios illustrate the D-A/D model's ability to "shape" the battle space to the defender's advantage using visible platforms in the first stage, and then to exploit the secrecy of hidden platforms for maximum benefit. Model instances are mixed-integer programs with up to 14,000 constraints and 12,000 variables. In each case, an optimal or near-optimal search plan coordinates the actions of multiple, heterogeneous ASW platforms to protect an HVU from an intelligent enemy.

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# LIST OF ACRONYMS

(This is the lexicon of anti-submarine warfare. Not all of these terms appear herein, but they are essential to deciphering our references.)

| | |
|---|---|
| 4W | Four-Whiskey Grid |
| | |
| ACINT | Acoustic Intelligence |
| ADCAP | Advanced Capability Torpedo (U.S. Navy) |
| AH | Acoustic Homer |
| AIP | Air-Independent Propulsion |
| ARCI | Acoustic Rapid Commercial-Off-The Shelf Insertion |
| ASCM | Anti-Ship Cruise Missile |
| ASPECT | Active System Performance Estimate Computer Tool |
| ASROC | Anti-Submarine Rocket |
| ASUW | Anti-Surface Warfare |
| ASW | Anti-Submarine Warfare |
| ASWC | Anti-Submarine Warfare Commander |
| ASWMP | Anti-Submarine Warfare Mission Planning |
| ATT | Anti-Torpedo Torpedo |
| AUV | Autonomous Underwater Vehicle |
| | |
| BB | Broadband |
| BG | Battle Group |
| | |
| CCD | Closed Cycle Diesel |
| CG | Cruiser (U.S. Navy *Ticonderoga*-class) |
| CM | Countermeasure(s) |
| CNO | Chief of Naval Operations |
| COA | Course of Action |
| COMINT | Communications Intelligence |
| COMSUBFOR | Commander, Submarine Forces |
| CONOPS | Concept of Operations |
| COTS | Commercial-Off-The-Shelf |
| CTF | Carrier Task Force |
| CVBG | Aircraft Carrier Battle Group |
| CVN | Aircraft Carrier (nuclear) |
| CVOA | Aircraft Carrier Operating Area |
| CZ | Convergence Zone |
| | |
| DD | Destroyer (U.S. Navy *Spruance*-class) |
| DDG | Guided Missile Destroyer (U.S. Navy *Arleigh Burke*-class) |
| DESRON | Destroyer Squadron |
| DP | Direct Path |

| | |
|---|---|
| EER | Extended Echo Ranging |
| ELINT | Electronic Intelligence |
| ESM | Electronic Surveillance Measures |
| FFG | Frigate (U.S. Navy *Perry*-class) |
| G-TAMP | Game-Theoretic ASW Mission Planner |
| HF | High-Frequency |
| HS | Sea-Based ASW Helicopter on Aircraft Carrier |
| HSL | Sea-Based ASW Helicopter on Surface Combatant |
| HVU | High-Value Unit |
| HWT | Heavyweight Torpedo |
| ISR | Intelligence, Surveillance, Reconnaissance |
| IUSS | Integrated Undersea Surveillance System |
| LAMPS | Light Airborne Multi-Purpose System (Helicopter) |
| LAS | Large Area Search |
| LCS | Littoral Combat Ship |
| LF | Low Frequency |
| LFA | Low Frequency Active |
| LP | Linear Program |
| LWT | Lightweight Torpedo |
| MAD | Magnetic Anomaly Detection |
| MF | Medium-Frequency |
| MIP | Mixed Integer (Linear) Program |
| MOE | Measure(s) of Effectiveness |
| MPA | Maritime Patrol Aircraft |
| NB | Narrowband |
| NM | Nautical Miles |
| OPAREA | Operating Area |
| OPSEC | Operational Security |
| OTH | Over the Horizon |
| PCIMAT | Personal Computer-Based Interactive Multisensor Analysis Training System |
| $P^D$ | Probability of Detection |
| PDR | Periscope-Detection Radar |
| PMI | Prevention of Mutual Interference |

| | |
|---|---|
| ROE | Rules of Engagement |
| SLBM | Submarine-Launched Ballistic Missile |
| SLCM | Submarine-Launched Cruise Missile |
| SLOC | Sea Lines of Communication |
| SOSUS | Sound Surveillance System |
| SSBN | Ballistic Missile Submarine (nuclear) |
| SSGN | Guided Missile Submarine (nuclear) |
| SSK | Hunter-Killer Submarine (diesel) |
| SSN | Hunter-Killer Submarine (nuclear) |
| SSTD | Surface Ship Torpedo Defense |
| SURTASS | Surveillance Towed Array Sensor System |
| SVP | Sound Velocity Profile |
| TDA | Tactical Decision Aid |
| UAV | Unmanned Aerial Vehicle |
| UUV | Unmanned Undersea Vehicle |
| VP | Fixed Wing Land-Based ASW Patrol Aircraft |
| VS | Fixed Wing Sea-Based ASW Aircraft |
| WSM | Water Space Management |
| WH | Wake Homer |

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# EXECUTIVE SUMMARY

A modern diesel-electric submarine (SSK) employs air-independent propulsion and lethal, "smart" weapons such as wake-homing torpedoes and anti-ship cruise missiles. Even a small country with a modest military force can purchase such an SSK from Russia or Germany for a relatively low price, and effectively challenge the U.S. Navy's ability to control and project power from the sea.

The U.S. Navy faces the possibility of conflict with an adversary armed with these SSKs. For instance, Iran, under worldwide pressure to abandon its nuclear ambitions, has recently threatened to use its small, but capable, submarine force to close the Strait of Hormuz, through which passes 40 percent of the world's oil. And, in October 2006, a Chinese *Song*-class SSK surfaced within weapons range of the *USS Kitty Hawk*, proving that even an antiquated diesel submarine can pose a threat to a U.S. aircraft carrier.

To counter this threat, the U.S. Navy employs surface, sub-surface, and airborne platforms to protect high-value units (HVUs) such as aircraft carriers from attack. In order to detect hostile submarines, these platforms use acoustic technology, which includes active and passive sonar in the form of hull-mounted and towed arrays, air-dropped sonobuoys, and dipping sonar, along with non-acoustic technology, such as periscope-detection radar and magnetic-anomaly detection.

The U.S. Navy uses several planning systems to help employ its anti-submarine warfare (ASW) platforms. Systems such as the Sonar Tactical Decision Aid (STDA), ASW Screen Planner Tactical Decision Aid, and Active Sensor Performance Estimate Computer Tool (ASPECT) focus on tactical details only: they do not coordinate or optimize the actions of multiple platforms. A fourth planning tool, the Operational Route Planner (ORP), attempts to optimize search-route planning with a heuristic, but models only a randomly behaving, "dumb" attacker. None of these models prescribes an optimal, coordinated, defense plan to protect an HVU from attack by an enemy who intelligently reacts to our defensive preparations.

To address the obvious shortcomings of current ASW mission-planning tools, we propose a new "Game-Theoretic ASW Mission Planner" (G-TAMP) as an operational-level planning aid for the pre-positioning of ASW platforms. The goal is to suggest how an ASW Commander might optimally coordinate multiple, heterogeneous defenders to protect an HVU from one or many intelligent, hostile SSKs.

Under reasonable assumptions, we express the enemy's course of action as a mathematical optimization to minimize the probability of detection of his SSKs, and the defender's course of action as an optimization to maximize the probability of detecting them. The defender employs "visible" platforms easily observed by the enemy, such as surface ships and platforms using active sonar, and "secret" platforms such as friendly submarines using passive sonar.

We first present a defender-attacker model (a two-stage Stackelberg game) in which the defender tasks platforms, and the enemy observes and reacts to these defenses; however, this model is appropriate only if all of the defender's platforms are visible. Therefore, G-TAMP improves on this by using a defender-attacker/defender (D-A/D) model that adds a final "defender stage" to task secret platforms unknown to the enemy. When the defender possesses platforms that can choose to use passive sonar (for secrecy) or active sonar (for increased detection ranges), the model prescribes the optimal sensor mode, in effect quantifying the value of secrecy for the defender.

The resulting mathematical model is a mixed integer linear program (MIP) that assigns each platform an area, time on station, and sensor mode. This mathematical program can be solved by commercial optimization software using traditional techniques; we also present a decomposition that accelerates solution times.

Five scenarios illustrate the power of our model. In each case, an optimal or near-optimal search plan coordinates the actions among each of the defender's ASW platforms. These scenarios illustrate G-TAMP's ability to "shape" the battle space to our advantage using visible platforms in the model's first stage, and then to exploit the secrecy of our hidden platforms as we "game" with the enemy in second stage.

# ACKNOWLEDGMENTS

First, I have to thank my incredible team of thesis advisors: Distinguished Professor Jerry Brown and Professor Kevin Wood, for their tireless instruction, seemingly endless patience, and constant encouragement to pursue my vision; and Senior Lecturer Jeff Kline, for giving me the original idea and letting me run with it. I am also extremely grateful to Distinguished Professor Emeritus Al Washburn, one of the true masters of the art of anti-submarine warfare, for inspiring the initial approach to this problem and providing sound advice along the way, and to Associate Professor Matt Carlyle, who always left his door open and made time for me in his busy schedule. This experience has not only been enlightening and educational, but also tremendously enjoyable and rewarding—it has been an honor to work with all of you. Finally, I have to thank my amazing wife, Christy, for sharing in my enthusiasm and encouraging me to dedicate my time to this thesis, even when I had to work late.

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# I. INTRODUCTION

This thesis develops a new, operational-level, mission-planning aid for anti-submarine warfare. This planning aid allows a battle group commander to optimally position and assign missions to his ASW platforms for protection of an aircraft carrier or other high-value units (HVUs) by maximizing the effectiveness of submarine search, while considering the enemy's intelligent response to these defensive actions.

This chapter explains how a modern submarine, in possession of an enemy willing to use it, poses a real threat to U.S. warships, and how the U.S. Navy plans to counter this threat. The chapter also describes the deficiencies of current ASW mission-planning tools, which helps to motivate the research described in this thesis.

## A. THE SUBMARINE THREAT

### 1. The Modern Submarine

Modern submarines, both nuclear and diesel-electric, pose a significant threat to the U.S. Navy and its ability to control and project power from the sea. For over a century, the submarine has been the ideal asymmetric weapon with which a weaker navy can successfully challenge a powerful adversary, because a submarine's inherent stealth gives it the unique ability to conduct surprise attacks and effectively evade counter-detection. Innovation has further tipped the scales in favor of the modern submarine, as new air-independent propulsion (AIP) technologies allow diesel-electric submarines (SSKs), which once had limited endurance, to remain submerged for weeks (Jane's 2005); Figure 1 illustrates some modern submarine technology.

Additionally, advances in weapons systems have greatly enhanced the lethality of modern submarines:

> Modern sensors and weapons pose a grave threat to U.S. and friendly naval and merchant forces. High-speed, wake-homing, and other advanced torpedoes are now available for open purchase. Submarines can also be armed with anti-ship cruise missiles, which can be launched while the submarines are submerged, and they can plant naval mines…, which have damaged or sunk more U.S. Navy ships since the end of WWII than any other weapon (Task Force ASW 2005).

**Figure 1. Modern submarine technology**

The German-made 212A SSK (from Jane's 2008a) employs a fuel cell for submerged propulsion, while other AIP boats use a Stirling closed-cycle diesel engine, shown right (from BAE 2007). Either technology allows for up to 14 days of submerged operations at 5 knots.

As a result, even a single submarine possesses the stealth and firepower necessary to penetrate a battle group's defenses and damage or sink a U.S. Navy warship.

### 2. Submarine Forces of the World

In recent years, submarines have become readily available to any country that desires a formidable naval force, as countries such as Russia and Germany have made submarine exports, equipped with the latest in AIP technology and smart weapons, available to the world market. The modest cost of a Russian *Kilo*-class submarine ($200 million each) allows many countries the opportunity to buy one of the quietest SSKs in existence. Iran, for example, has purchased three *Kilos*, and China has purchased 12, eight of which were added in just a two-year period (Jane's 2008b). Table 1 outlines worldwide submarine inventories.

**Modern German Conventional Subs**

| | |
|---|---|
| -Argentina | -Indonesia |
| -Brazil | -Israel |
| -Chile | -Norway |
| -Colombia | -Peru |
| -Denmark | -S. Africa |
| -Ecuador | -S. Korea |
| -Germany | -Turkey |
| -Greece | -Venezuela |
| -India | |

**Other Modern Western Conventional Subs**

| | |
|---|---|
| -Australia | -Netherlands |
| -Brazil | -Norway |
| -Canada | -Pakistan |
| -Chile | -Singapore |
| -Egypt | -S. Africa |
| -Israel | -Spain |
| -India | -Sweden |
| -Italy | -Taiwan |
| -Japan | -Thailand |
| -Malaysia | |

**Modern Russian Conventional Subs**

| | |
|---|---|
| -Algeria | -Poland |
| -China | -Romania |
| -India | -Russia |
| -Iran | |

**Non-Modern Conventional Subs**

| | |
|---|---|
| -Bulgaria | -Poland |
| -China | -Taiwan |
| -Egypt | -Turkey |
| -India | -Ukraine |
| -Libya | -Yugoslavia |
| -N. Korea | |

**Nuclear Subs**

| | |
|---|---|
| -U.S. | -France |
| -U.K. | -China |
| -Russia | -India |

**Countries with Large Inventories**

| | |
|---|---|
| -China: | 80+, including *Kilo*, *Song*, and nuclear and AIP boats |
| -U.S.: | 53 SSN, 4 SSGN, 14 SSBN |
| -N. Korea: | 23 *Romeo*, 65 coastal |
| -Iran: | 3 *Kilo*, 7 coastal |

**Table 1. Countries with submarine forces**

More than 40 countries have submarines (after Benedict 2006), including several countries with large inventories. (SSN, SSGN and SSBN denote nuclear fast-attack, guided-missile, and ballistic missile submarines, respectively.)

## 3. Future Conflict

The U.S. Navy faces the near-term possibility of conflict with an adversary armed with submarines. Iran, for example, has recently announced the production of a new line of submarines, and Iran's Defense Minister has proclaimed their intent to use these, along with their small, but capable, existing submarine force, to retaliate against any military attack by closing the Strait of Hormuz (Boston Globe 2008). Considering Iran's defiant pursuit of a nuclear program, and the fact that the Strait of Hormuz carries 40 percent of the world's oil, the U.S. cannot afford to ignore this threat. Holland (1991) explains the effect submarines would have: "In a conflict with less than a superpower, public or political patience will run thin concerning losses or delays by submarines. The magnitude of the political catastrophe arising from the torpedoing of an aircraft carrier in a limited conflict can hardly be overestimated."

China demonstrated its underwater-warfare capabilities in October 2006, when a *Song*-class SSK surfaced within weapons range of the *USS Kitty Hawk*. No weapons were fired, but this proves that even an antiquated diesel submarine can pose a threat to a U.S. aircraft carrier.

## B. ANTI-SUBMARINE WARFARE PLATFORMS AND SYSTEMS

The U.S. Navy uses the ASW platforms described below to search for, detect, classify, track, and prosecute hostile submarines. These platforms, when included in the battle group, provide protection for HVUs as directed by the Joint Maritime Component Commander or Area ASW Commander (see DOD 2006). This thesis develops a planning aid for the operational employment of these platforms.

### 1. Surface Platforms

Surface ships such as the *Ticonderoga*-class cruiser, *Arleigh Burke*-class destroyer, and *Perry*-class frigate (see Figure 2) perform the bulk of ASW missions. These ships employ active sonar, which uses transmitted acoustic pulses and subsequent echo returns to localize contacts, and passive sonar, which relies only on listening. Their sonar systems use both hull-mounted and towed-array sensors. When part of an aircraft carrier battle group (CVBG), these platforms act as a protective "screen" for the carrier, a defensive perimeter that deters submarine attack by employing these sensor systems, weapons such as anti-submarine torpedoes, and countermeasures such as decoys and anti-torpedo torpedoes.

**Figure 2. ASW-capable surface platforms**

A frigate (*USS Nicholas*, FFG 47, left), guided-missile destroyer (*USS Bainbridge*, DDG 96, top right), and cruiser (*USS Normandy*, CG 60, bottom right) are all capable of ASW search and prosecution (from Jane's 2008c).

### 2. Airborne Platforms

The U.S. Navy employs both fixed- and rotary-wing aircraft for ASW; Figure 3 shows two examples. The SH-60 helicopter, operated from cruisers, destroyers, and frigates, can drop sonobuoys capable of active or passive operation, and can "dip sonar," i.e., can lower a sonar sensor into the ocean on a cable. The land-based P-3 airplane can deploy its sonobuoy inventory in various patterns to enhance search effectiveness, and is capable of extended echo ranging, which uses the sound from detonating small explosive charges to locate submarines. In addition to sonar sensors, both the SH-60 and P-3 are equipped with surface-search periscope-detection radar that can detect and localize an exposed submarine periscope or mast. Both platforms employ lightweight torpedoes for attacks. The next-generation successor to the P-3, the P-8, deploys in 2013, and will improve upon the P-3's capabilities, including the ability to control unmanned aerial vehicles for submarine search.

**Figure 3. Airborne ASW platforms**

The SH-60 Seahawk helicoper (left) and P-3 Orion airplane (right) perform large-area ASW searches (from Jane's 2008d).

### 3. Sub-Surface Platforms

A CVBG typically contains two fast-attack submarines that serve as sub-surface ASW platforms. The *Los Angeles*, *Seawolf,* and *Virginia* classes of submarine (see Figure 4) are all equipped with active and passive, hull-mounted and towed-array sonar systems. These submarines often provide the most effective sonar search capability due

to their low self noise, inherent stealth, and ability to vary search depth to capitalize on the ocean's local acoustic conditions. They are armed with long-range, wire-guided, heavyweight torpedoes for engaging enemy combatants.

**Figure 4. *Los Angeles*-class submarine**
Each carrier battle group typically includes two fast-attack submarines for sub-surface ASW support, such as the *Los Angeles*-class submarine *USS Asheville*, SSN 758, shown here (from Jane's 2008e).

## C. CURRENT PLANNING TOOLS

The U.S. Navy currently employs several tools for planning ASW missions. Each tool may succeed in its intended goal, but none optimally coordinates the actions of multiple ASW platforms to protect an HVU, and none considers an enemy's intelligent response to our defensive preparations.

### 1. Personal Computer-Based Interactive Multisensor Analysis Training System (PCIMAT) and Sonar Tactical Decision Aid (STDA)

The Personal Computer-Based Interactive Multisensor Analysis Training System (PCIMAT), also known as the Sonar Tactical Decision Aid (STDA), is the premier ocean acoustic analysis and planning tool available on all ASW platforms, including surface ships, aircraft, and submarines. The Space and Naval Warfare Systems Command originally developed this system as a training tool, but it has since evolved into a tactical decision aid for ASW platforms. Although employable from a laptop computer (as PCIMAT), the latest technology integrates its capabilities with a platform's entire fire-

control suite (and is then called STDA). PCIMAT offers planners the ability to plan a sonar search based on measurements of the local ocean environment, or ocean conditions can be retrieved from a worldwide, historical database, downloaded in message traffic that uses up-to-date satellite imagery, or forecast for up to 48 hours in advance using the Navy's new Global Navy Coastal Ocean Model (SPAWAR 2008).

PCIMAT provides ocean temperature estimates, which affect sensor performance, as well as a visual representation of estimated sound propagation paths (i.e., "ray tracing"), which helps planners decide where to place sensors to enhance search effectiveness. The newest version of PCIMAT adds a mission-planning module that analyzes the acoustic conditions at a user-designated geographic location and provides a graphical representation of a platform's effective search range and its vulnerability to counter-detection by an enemy.

### 2. ASW Screen Planner Tactical Decision Aid (TDA)

The ASW Screen Planner Tactical Decision Aid (TDA) helps the ASW Commander to plan ASW screens. The planner specifies a threat submarine (from a database) and assigns available ASW platforms to sectors surrounding the HVU, and the TDA calculates the probability of detecting the enemy submarine given that it transits a particular sector. (A bearing spread and range window define each sector, for example, all bearings from 000 to 020 degrees at ranges between 10,000 and 20,000 yards.) The planner then iteratively and manually assigns platforms to sectors until he creates a solution with an acceptable probability of detection (SWDG 2004).

### 3. Active System Performance Estimate Computer Tool (ASPECT)

The Active System Performance Estimate Computer Tool (ASPECT) aids P-3 aircraft in maximizing the effectiveness of active sonobuoy search. The planner manually specifies several sonobuoy patterns (such as a grid, circle, or "V" arrangement) and a ping rate (which determines how often and in what sequence each buoy transmits pings). ASPECT then uses a Monte Carlo simulation to generate approximately 500 random submarine tracks in the area to be searched, and simulates the entire mission for

each track and each sonobuoy pattern and ping rate combination. The planner then chooses the solution with the highest probability of detection, which ASPECT estimates from the percentage of target tracks detected (FAST 2006).

#### 4. Operational Route Planner (ORP)

The Operational Route Planner (ORP) models the "area search problem," whose goal is to best route search platforms, at the tactical level, to detect an SSK that may be present in a designated geographic area. ORP uses a genetic algorithm to provide heuristically optimized search plans for multiple ASW defenders. The planner specifies available platforms and assigns patrol regions, and the solution provides detailed, coordinated track plans for the searchers. ORP simulates the attacker's actions using Monte Carlo simulation based on a probabilistic description of assumed enemy behavior and several user-selected rules (for example, the planner can specify that an SSK will change depth when it moves within a certain range of a defender) (Wagner Associates 2008).

These four currently employed systems offer several different ways to plan ASW missions, but none provides a means of optimally and synergistically coordinating the actions of multiple, heterogeneous platforms at the operational level in an effort to protect an HVU. Also, because these systems assume that the enemy ignores defensive preparations, or that he behaves randomly (or according to a simple set of rules), they cannot model a determined attacker who responds to our defenses with intelligent avoidance tactics.

# II. THE GAME-THEORETIC ASW MISSION PLANNER MODEL

To address the shortcomings of current ASW mission planning tools, we propose the Game-Theoretic ASW Mission Planner (G-TAMP), an operational-level planning aid for the coordinated mission tasking of heterogeneous ASW platforms in order to protect one, or more, designated HVUs from attack by hostile SSKs.

This chapter develops two models for optimizing defense plans. We first present a defender-attacker model (D-A) that has a "defender" locating ASW platforms to minimize the probability that an "attacker" reaches an HVU, while the attacker observes and responds to these preparations by routing his SSKs to maximize that probability. Having introduced the D-A model, we can more easily present the second model, which G-TAMP uses for its solutions: a defender-attacker/defender model (D-A/D) that adds a final stage to optimize the actions of "secret" defensive platforms i.e., those using passive sonar that the enemy cannot observe.

## A. ASW TERMINOLOGY

Throughout this section, terms in italic font define a formal model lexicon.

### 1. Geography

We divide the ocean into a Four-Whiskey (4W) grid, which the U.S. Navy commonly uses to partition the ocean and coordinate operations. 4W grids are stationary, and vary in cell size and overall dimension, depending on the region they cover; typical cell sizes range from 5 $nm^2$ to 10 $nm^2$. A grid cell is denoted by index $g$. Figure 5 shows an example of a 6×6 4W grid.

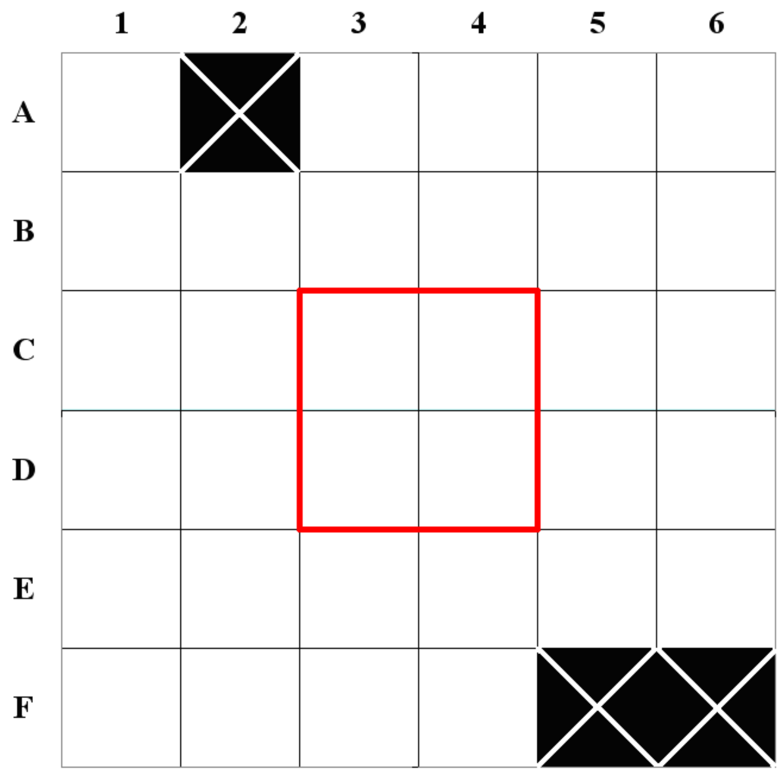


**Figure 5. 4W grid**

A 4W grid cell is identified by its row letter and column number, e.g., C3. Each black cell with a white "x" is impassable; all other cells are traversable. The boxed region denotes protected cells.

Each *traversable cell* is a candidate location through which an enemy SSK can pass and in which we can search using friendly platforms; *impassable cells*, which represent land or shoal waters, do not allow passage or search. We refer to each cell by an alpha-numeric row and column label, for example, "A1" is the cell in the upper left-hand corner of the grid.

A *protected cell* is a traversable cell from which an enemy SSK can conduct an attack on the HVU. Therefore, ASW defenders must take positions outside of the protected cells in order to prevent SSKs from entering. We assume that the HVU remains inside of the protected cells, for example, to support air operations. While, in reality, the HVU moves around within the region of protected cells, we assume that the enemy can conduct an attack from any protected cell.

Each traversable cell $g$ is connected to each adjacent traversable cell $g'$ by a directed *arc* $(g, g')$ in a network model. An arc represents a feasible move for an SSK in either the horizontal, vertical, or diagonal direction. For example, in Figure 5, an SSK could move from cell B1 to cells A1, B2, C1, or C2, but not to A2.

### 2. Measure of Effectiveness

Each defensive platform has an effective range at which it can detect an enemy submarine. Twice this range defines the platform's *sweep width*, which represents the detection potential of the sensor. The sweep width is twice this range because we assume the platform can make a detection equally well to its left and right. An alternative definition of sweep width is the area under the lateral range function $p(x)$ of the sensor, which is defined as "the probability that the target will be detected if its track relative to the searcher is a straight line infinitely long in both directions with closest point of approach $x$" (Washburn 2002, pg. 4-1).

To quantify the effectiveness of our search, we assume, as does Washburn (2007), that each traversable cell requires a certain amount of time for a given platform to search completely. A platform will search an area per unit time equal to its search speed times sweep width. Therefore, we define *coverage rate* as the fraction of a cell searched per unit time: $r = \dfrac{(sweep\ width)(speed)}{cell\ area}$. In reality, coverage rate depends on a platform's sensor system performance and crew proficiency, as well as environmental factors such as a cell's temperature distribution, sea state, sound-speed profile, bathymetry, sea-life concentration, shipping density, and ambient noise (see Urick 1983). Coverage rate also varies with the choice of active or passive sensor mode; therefore, a coefficient $r_{psg}$ specifies the coverage rate for platform $p$ as it searches with sensor mode $s$ in cell $g$.

The search *pressure* in a given cell is the amount of search effort applied to that cell, i.e., the sum over all platforms of the product of coverage rate and time spent searching in that cell. Cells with "high pressures" are well-searched, and therefore unattractive to the enemy.

### 3. Enemy Course of Action

An enemy SSK attempts to carry out an attack on the HVU by transiting from some location outside of the 4W grid to some protected cell, which is within weapons range of the HVU. To model this, we assume that each SSK transit begins at an artificial *start cell* $g^+$, i.e., a "super-source" that connects to all cells through which the SSK can enter the 4W grid, and we assume that each transit ends at an artificial *terminal cell* $g^-$, i.e., a "super-sink" that connects from all protected cells.

The attacker's goal is to route his SSKs from $g^+$ such that the probability that at least one SSK reaches $g^-$ is maximized. The attacker approximates this by planning routes for the SSKs that minimize total accumulated pressure along their paths, which equates to minimizing the expected number of detections (e.g., Washburn and Wood 1995; see also Appendix A).

### 4. Friendly Course of Action

Each friendly ASW *platform* can search for enemy SSKs in cells to which it is assigned. Platforms are subdivided into several types.

Each *visible platform* is observable to the enemy. Visible platforms include all surface ships, which are easily observed visually, acoustically, or by electronic surveillance measures, and any platform that employs active sonar, which an approaching SSK can easily detect. Enemy SSKs react to the pressures exerted by visible platforms.

A *secret platform* cannot be observed by the enemy. Secret platforms include friendly submarines employing passive sonar and passive sonobuoys deployed by aircraft. Enemy SSKs may know that a secret platform is present—for instance, a CVBG is normally supported by two submarines—but a secret platform's exact location, and therefore the pressure it exerts, remains unknown to the enemy.

Each *flexible platform* can decide whether to operate visibly or in secret. If a flexible platform chooses a mission with an active sensor mode, it gains in sensor performance, but becomes visible. Likewise, it can choose a passive sensor mode to hide

from the enemy, but it must accept a decrease in search effectiveness. A flexible platform will always decide to become either visible or secret, but never both.

Each *tethered platform*, such as a helicopter operated from a surface ship, must be able to return to its *base platform*; therefore, a tethered platform can perform missions only within its *tether range*.

Each *mission* assigns a platform to a group of contiguous cells to be searched, a time to patrol each cell, and a *sensor mode s* to use (*active* or *passive*). To ensure that a mission contains cells within reasonable proximity of each other, we view the 4W grid network as an undirected graph and require that the cells that make up a mission, along with connecting arcs, form a connected subgraph. We then enumerate each subgraph containing up to *n* cells, where *n* is the maximum number of cells that a platform can realistically search over the mission time horizon. Each such subgraph becomes a candidate mission if it meets the planner's requirements with respect to platform speed, range, the overall length of the mission, and pre-designated area restrictions (for example, submarines typically are assigned to limited water space to prevent conflicts).

The defender's goal is to assign mission tasking to platforms in order to maximize the pressure along the enemy's paths. Appendix A shows how maximizing total pressure corresponds to maximizing the probability of detection of all SSKs that might be attacking the HVU.

## B. A DEFENDER-ATTACKER (D-A) MODEL, SCREEN

We assume initially that the defender possesses only visible platforms. We further assume that these platforms deploy before the attacker begins any attack, and that both attacker and defender have the same information available. The attacker observes the pressures exerted over the entire mission time horizon, and then responds by routing his SSKs intelligently in order to minimize the accumulated pressure. (This is not a time-phased model, which would make sense only if we have some knowledge of when the attacker might begin an attack; we have no such knowledge.) Because of the sequential actions, this is a type of two-stage Stackelberg game, which we represent as a *defender-attacker* (D-A) model (Brown et al. 2006). Typical solutions to this D-A model create a

"screen" of defensive platforms around the HVU, so we call this model **SCREEN**. It provides a worst-case scenario in which the attacker knows our defensive plan completely. A detailed description follows.

### 1. Indices and Index Sets

| | |
|---|---|
| $p \in P$ | defending platforms |
| $p' \in P_{BASE} \subseteq P$ | set of base platforms for tethered platforms |
| $p'' \in P_{TETH} \subseteq P$ | set of platforms that are tethered to another platform, where $P_{BASE} \cap P_{TETH} = \varnothing$ |
| $(p', p'') \in PP_{TETH}$ | set defining each base and tethered platform pair |
| $m \in M$ | possible missions |
| $s \in S$ | sensor modes, with $S = \{s_1, s_2\} \equiv \{active, passive\}$ |
| $g \in G$ | 4W grid cells |
| $m \in M_{ps} \subseteq M$ | missions platform $p$ can perform with sensor mode $s$ |
| $g \in G_{pm} \subseteq G$ | cells patrolled by platform $p$ when executing mission $m$ |
| $(g, g') \in A$ | adjacency list specifying allowed SSK moves from grid cell $g$ to grid cell $g'$ |
| $g^+, g^-$ | artificial start cell and terminal cell for SSKs, respectively |

### 2. Data [Units]

| | |
|---|---|
| $r_{psg}$ | coverage rate of platform $p$ using sensor mode $s$ in grid cell $g$ [$\mathrm{hr}^{-1}$] |
| $time_{pm}$ | time on station for platform $p$ when executing mission $m$ [hr] |
| $n_{SUBS}$ | number of attacker submarines [submarines] |
| $\overline{n}_{PLATS}$ | maximum number of platforms that can search a given cell [platforms] |

| | |
|---|---|
| $range_{m'm''}$ | shortest straight-line distance between some cell in mission $m'$ and some cell in mission $m''$ [nm] |
| $teth\_range_{p''}$ | maximum range tethered platform $p''$ can travel from its base platform before beginning a mission [nm] |
| $u$ | maximum number of SSKs that can traverse any arc $(g,g') \in A$ [submarines] |
| $trans_p$ | time required for platform $p$ to transit a grid cell [hr] |

### 3. Variables [Units, if applicable]

| | |
|---|---|
| $Y_{gg'}$ | expected number of SSKs that travel arc $(g,g')$ [submarines] |
| $R_{pms}$ | 1 if platform $p$ executes mission $m$ using sensor mode $s$, 0 otherwise |
| $X_{pmsg}$ | time that platform $p$ spends executing mission $m$ using sensor mode $s$ in grid cell $g$ [hr] |

Note: Bold roman letters represent vectors, e.g., **Y**, **R**, **X**.

### 4. Generic Constraint Sets (described in detail subsequently)

| | |
|---|---|
| $(\mathbf{R},\mathbf{X}) \in \mathcal{RX}$ | limits on employment of defensive platforms |
| $\mathbf{Y} \in \mathcal{Y}$ | routing constraints on the attacker |

## 5. Max-Min Optimization of Pressure along Attacker's Path [Dual Variables]

**SCREEN**

$$\max_{(\mathbf{R},\mathbf{X})\in\mathcal{RX}} \left[ \begin{array}{lll} \min\limits_{\mathbf{Y}} & \sum\limits_{(g,g')\in A} \sum\limits_{\substack{p\in P, m\in M_{ps} \\ s\in S|g'\in G_{pm}}} r_{psg'} X_{pmsg'} Y_{gg'} & (a0) \\ \text{s.t.} & \sum\limits_{g'|(g,g')\in A} Y_{gg'} - \sum\limits_{g'|(g',g)\in A} Y_{g'g} = \left\{ \begin{array}{lll} +n_{SUBS} & g=g^{+} & [\alpha_{g^{+}}] \\ 0 & g\in G-\{g^{+},g^{-}\} & [\alpha_{g}] \\ -n_{SUBS} & g=g^{-} & [\alpha_{g^{-}}] \end{array} \right\} & (a1) \\ & 0\le Y_{ij}\le u \qquad\qquad\qquad\qquad [\beta_{gg'}] & (a2) \end{array} \right]$$

The attacker's objective (*a0*) is to minimize the accumulated pressure along the paths of his SSKs as they travel towards the HVU. Constraints (*a1*) enforce a balance of flow through each traversable cell, that is, each SSK entering a cell must leave that cell, except at $g^{+}$ and $g^{-}$. Constraints (*a2*) limit the number of SSKs that can traverse between two given cells. Without these constraints, an optimal solution for the attacker might send all SSKs along the same path, which seems unlikely for a stealthy adversary who is attempting to maximize the probability that at least one of his SSKs reaches a protected cell. Together, constraints (*a1*) and (*a2*) form the attacker's feasible region $\mathbf{Y}\in\mathcal{Y}$.

The quantity $\sum\limits_{\substack{p\in P, m\in M_{ps} \\ s\in S|g'\in G_{pm}}} r_{psg'} X_{pmsg'}$ represents the pressure in cell $g'$. Therefore, if the attacker chooses to send some of his SSKs through $g'$, he accumulates a cost of $\sum\limits_{\substack{p\in P, m\in M_{ps} \\ s\in S|g'\in G_{pm}}} r_{psg'} X_{pmsg'} Y_{gg'}$. By minimizing (*a0*), the attacker chooses paths with minimum total pressure.

### 6. Limits on Defender's Actions

The defender's actions are limited by the following:

$$\sum_{m \in M_{ps}, s \in S} R_{pms} \leq 1 \qquad \forall p \in P \qquad (d1)$$

$$\sum_{g \in G_{pm}} X_{pmsg} \leq time_{pm} R_{pms} \qquad \forall p \in P, m \in M_{ps}, s \in S \qquad (d2)$$

$$X_{pmsg} \geq trans_p R_{pms} \qquad \forall p \in P, m \in M_{ps}, s \in S, g \in G_{pm} \qquad (d3)$$

$$\sum_{\substack{p \in P, m \in M_p \\ s \in S | g \in G_{pm}}} R_{pms} \leq \overline{n}_{PLATS} \qquad \forall g \in G \qquad (d4)$$

$$R_{p''m''s''} \leq \sum_{\substack{(p',p'') \in PP_{TETH}, \\ m' \in M_{p's'}, s' \in S | \\ range_{m'm''} \leq \\ teth\_range_{p''}}} R_{p'm's'} \qquad \forall p'' \in P_{TETH}, m \in M_{p''s''}, s'' \in S \qquad (d5)$$

$$R_{pms} \in \{0,1\} \qquad \forall p \in P, m \in M_{ps}, s \in S \qquad (d6)$$

$$X_{pmsg} \geq 0 \qquad \forall p \in P, m \in M_{ps}, s \in S, g \in G \qquad (d7)$$

Together, constraints (*d1*) through (*d7*) form the defender's feasible region $(\mathbf{R}, \mathbf{X}) \in \mathcal{RX}$.

Constraints (*d1*) require that each platform choose a single mission; constraints (*d2*) require that each platform use only the available time on station for the chosen mission; constraints (*d3*) require that a platform spend at least the amount of time required to transit a cell in each cell of the chosen mission; constraints (*d4*) limit the number of platforms that can occupy a single grid cell; constraints (*d5*) require that each tethered platform choose a mission within the tether range of its base platform; constraints (*d6*) require binary decisions; and constraints (*d7*) enforce non-negativity of the time spent on station.

### 7. Improvements to SCREEN

**SCREEN** generally provides solutions that optimally place defensive platforms in cells immediately surrounding the protected region. As a result, defensive platforms do not patrol more distant cells at all, and so these distant cells receive zero pressure. This implies that the attacker could choose an indirect path that meanders around the 4W grid

before moving towards a protected cell. However, we wish to model an attacker that takes the shortest route possible, unless he can avoid high-pressure cells by taking a longer one. This realistically approximates the behavior of a diesel-electric submarine, as the survival of an SSK during combat depends on its remaining battery life. Therefore, we introduce the parameter *battery*, which adds a small pressure cost for every arc traversed by an SSK.

Occasionally, the defender has multiple optimal solutions and **SCREEN** will recommend placing defending platforms farther out from the perimeter of protected cells than necessary. In reality, a battle group commander wants to keep as many platforms as close to the HVU as possible for air defense considerations. Therefore, we introduce a "tie-breaking" term to modify coverage rate, which reduces the coverage rates in more distant cells by a small amount.

Our formulation then includes the following additional terms:

| | |
|---|---|
| *battery* | battery penalty for traversing a single arc [pressure] |
| $dist_g$ | distance from cell $g$ to nearest protected cell [nm] |
| $d$ | weighting factor for screen distance penalty [$1/nm \cdot hr$] |

We then include these terms in the attacker's objective function (*a0*), which takes the form:

$$\min_{\mathbf{Y}\in\mathcal{Y}} \sum_{(g,g')\in A} \left( battery + \sum_{\substack{p\in P, m\in M_{ps} \\ s\in S | g'\in G_{pm}}} (r_{psg'} - d \cdot dist_{g'}) X_{pmsg'} \right) Y_{gg'} \qquad (a0)$$

The magnitude of *battery* is much less than the pressure in searched cells, so it induces no unreasonable behavior in the attacker's solution; that is, if the attacker can circumvent the defender by taking a longer path, he will choose it; otherwise he takes the shortest path possible.

Likewise, because $d$ is also a small number, the distance penalty produces no unreasonable defender behavior. If the defender benefits from moving the screen away from the protected cells, he will do so; otherwise he places defenders as close to the protected cells as possible.

For ease of exposition, we define

$$r'_{psg'} = (r_{psg'} - d \cdot dist_{g'}).$$

Henceforth, when we refer to pressure in a given cell, that implies the quantity

$$\sum_{\substack{p\in P, m\in M_{ps} \\ s\in S | g'\in G_{pm}}} r'_{psg'} X_{pmsg'} .$$

When we refer to pressure along an SSK path, that implies the sum

$$\sum_{(g,g')\in path} \left( battery + \sum_{\substack{p\in P, m\in M_{ps} \\ s\in S | g'\in G_{pm}}} r'_{psg'} X_{pmsg'} \right) Y_{gg'} .$$

"**SCREEN**" then refers to the revised model

**SCREEN**

$$\max_{(\mathbf{R},\mathbf{X})\in\mathcal{RX}} \left[ \begin{array}{lll} \min\limits_{\mathbf{Y}} & \displaystyle\sum_{(g,g')\in A} \left( battery + \sum_{\substack{p\in P, m\in M_{ps} \\ s\in S | g'\in G_{pm}}} r'_{psg'} X_{pmsg'} \right) Y_{gg'} & (a0) \\ \text{s.t.} & \displaystyle\sum_{g'|(g,g')\in A} Y_{gg'} - \sum_{g'|(g',g)\in A} Y_{g'g} = \left\{ \begin{array}{lll} +n_{SUBS} & g = g^{+} & [\alpha_{g^{+}}] \\ 0 & g\in G-\{g^{+},g^{-}\} & [\alpha_{g}] \\ -n_{SUBS} & g = g^{-} & [\alpha_{g^{-}}] \end{array} \right\} & (a1) \\ & 0 \le Y_{ij} \le u \qquad\qquad [\beta_{gg'}] & (a2) \end{array} \right]$$

## 8. Conversion of SCREEN to a Mixed-Integer Program, SCREEN$_{\mathbf{MIP}}$

We can temporarily fix **R** and **X**, take the dual of the inner, minimizing linear program and then release **R** and **X.** This creates a monolithic, mixed-integer linear program (MIP), **SCREEN**$_{\mathbf{MIP}}$.

**SCREEN**$_{\mathbf{MIP}}$

$$\max_{\substack{\mathbf{R},\mathbf{X},\\ \boldsymbol{\alpha},\boldsymbol{\beta}}} n_{SUBS}\,\alpha_{g^+} - n_{SUBS}\,\alpha_{g^-} - u\sum_{(g,g')\in A}\beta_{gg'} \qquad (b0)$$

$$\text{s.t.}\quad \alpha_g - \alpha_{g'} - u\beta_{gg'} \le battery + \sum_{\substack{p\in P, m\in M_{ps}\\ s\in S|g'\in G_{pm}}} r'_{psg'}X_{pmsg'} \quad \forall (g,g')\in A \qquad (b1)$$

$$\alpha_{g^+} = 0 \qquad (b2)$$

$$\beta_{gg'} \ge 0 \qquad \forall (g,g')\in A \qquad (b3)$$

$$(\mathbf{R},\mathbf{X}) \in \mathcal{RX}$$

Given an optimal defense plan $\mathbf{X}^*$ from **SCREEN**$_{\mathbf{MIP}}$, we can recover an optimal attacker solution by fixing $\mathbf{X}^*$ in **SCREEN** and solving the resulting linear program, which is a pure, totally unimodular, network model (e.g., Ahuja et al., 1993, pp. 448-449); therefore, the solution obtained, $\mathbf{Y}^*$, will be integral.

However, due to the nature of the screening solution, the attacker may have multiple optimal solutions, and recovering a single solution (falsely) implies that the defender knows the attacker's path with certainty. This may not create difficulties when the defender employs only visible platforms, but it could lead to incorrect results if we were to add a third, sequential-decision stage to model secret defensive platforms. In particular, such a model might act as if the defender, in the third stage, knows the attacker's exact path or paths: this is unreasonable. To avoid this, we use a three-stage, game-theoretic model that incorporates both sequential and simultaneous play.

## C. A DEFENDER-ATTACKER/DEFENDER (D-A/D) MODEL, SSCREEN

The Stackelberg **SCREEN** model provides the ability to optimally and synergistically create a screen of pre-positioned ASW defenses around an HVU to deter an enemy attack, which is a significant improvement over existing planning tools. However, it assumes that the attacker observes all defensive preparations, and this may be too conservative for the defender.

In addition to the visible platforms used in a screen, ASW planners can employ secret platforms whose positions are not known to the enemy. Typically, when detecting quiet submarines operating on battery, an active sensor mode provides increased detection range and bearing resolution, but it also reveals the originating platform's location. By employing secret platforms that use passive sonar, can we leverage their secrecy to create a better solution? To answer this question, we present the following three-stage model, **SSCREEN**.

**SSCREEN** could be modeled as a defender-attacker-defender problem with strict sequential play, i.e., as a three-stage Stackelberg game: the maximizing defender would task his visible platforms in the first stage, the minimizing attacker would route his SSKs optimally in the second stage, and the maximizing defender would assign missions to his secret platforms based on the SSKs' paths. Such a model has been formulated and solved, but the solutions it yields are unrealistic. In particular, if an SSK has only a single optimal path, then one or more secret defensive platforms will simply lie in wait and kill it with high probability. But, if the SSK does not have the exact same view of the environment the attacker does, these secret defensive platforms may be avoided, simply by chance.

The solution to this dilemma emerges naturally when one realizes that the attacker is likely to have some knowledge of the defender's secret platforms. For instance, a CVBG is normally supported by two submarines, and if the attacker cannot detect one or both, then he knows, with a high level of certainty, that one or two secret defensive platforms are in the vicinity. As another example, suppose that the attacker has detected a P-3 aircraft in the vicinity of the CVBG, but detects no active sonobuoys that the P-3

might deploy. He may reasonably conclude that the aircraft has secretly deployed some passive sonobuoys around the battle group.

Therefore, the inner attacker/defender model in **SSCREEN** may be viewed as a two-person, zero-sum game (TPZSG) with simultaneous play: the attacker will need to randomize his attack strategy and the defender, in the last stage, will need to randomize the mission assignments for his secret platforms. For this reason, we define **SSCREEN** as a defender-attacker/defender (D-A/D) model, where the hyphen indicates sequential play, and the forward slash indicates simultaneous play. The random attack strategy will be influenced by the "deterministic" first stage of the model, but attack variables **Y** can now be viewed as representing (continuous) probabilities. The defender's third-stage variables **Q** will then define the probability that a secret defensive platform is assigned to a given mission, or a mixed strategy. A detailed description of this model follows.

### 1. Additional Indices and Index Sets

| | |
|---|---|
| $P_{VIS} \subseteq P$ | set of defending platforms potentially visible to the attacker |
| $P_{SECRET} \subseteq P$ | set of defending platforms effectively invisible to the attacker, where $P_{VIS} \cap P_{SECRET} = \varnothing$ |
| $P_{FLEX} \subseteq P$ | set of defending platforms that can choose to operate in an active or passive mode |

$P_{VIS+} = P_{VIS} \cup P_{FLEX}$

$P_{SECRET+} = P_{SECRET} \cup P_{FLEX}$

### 2. Additional Variables [Units]

| | |
|---|---|
| $Q_{pms}$ | probability that platform $p \in P_{SECRET}$ executes covert mission *m* using sensor mode *s* [probability] |
| $W_{pmsg}$ | expected amount of time platform $p \in P_{SECRET}$ spends executing covert mission *m* using sensor mode *s* in cell *g* [hr] |

**3. Generic Constraint Sets (described in detail subsequently)**

| | |
|---|---|
| $(\mathbf{Q},\mathbf{R},\mathbf{X}) \in \mathcal{QRX}$ | limits on employment of visible defensive platforms, including which flexible platforms will be used as secret ones |
| $(\mathbf{Q},\mathbf{W}) \in \mathcal{QW}$ | limits on employment of secret defensive platforms |
| $\mathbf{Y} \in \mathcal{Y}$ | routing constraints on the attacker |

**4. Max-Min/Max Optimization of Pressure along Attacker's Path**

Formulated as a tri-level integer linear program, the model takes the form

**SSCREEN**

$$\max_{(\mathbf{Q},\mathbf{R},\mathbf{X})\in\mathcal{QRX}} \left[ \min_{\mathbf{Y}\in\mathcal{Y}} \left[ \max_{(\mathbf{Q},\mathbf{W})\in\mathcal{QW}} \sum_{(g,g')\in A} f_{g'}(\mathbf{W},\mathbf{X}) Y_{gg'} \right] \right] \quad (a0')$$

where

$$f_{g'}(\mathbf{W},\mathbf{X}) \equiv battery + \sum_{\substack{p\in P_{VIS},m\in M_{ps} \\ s\in S|g'\in G_{pm}}} r'_{psg'} X_{pmsg'} + \sum_{\substack{p\in P_{FLEX}, \\ m\in M_{ps}|g'\in G_{pm}}} r'_{p,s_1,g'} X_{pm,s_1,g'}$$
$$+ \sum_{\substack{p\in P_{FLEX}, \\ m\in M_{ps}|g'\in G_{pm}}} r'_{p,s_2,g'} W_{pm,s_2,g'} + \sum_{(g,g')\in A} \sum_{\substack{p\in P_{SECRET},m\in M_{ps} \\ s\in S|g'\in G_{pm}}} r'_{psg'} W_{pmsg'} \quad (a3)$$

As before, the defender pre-positions his visible platforms, and the attacker chooses the path of minimum pressure. However, the defender now has the ability to position additional, secret platforms to increase the pressure along the defender's path. We treat **Q**, **W** and **Y** as continuous, so that their values suggest mixed strategies.

### 5. Limits on Defender's Visible Platforms

The outer maximization problem has feasible region $(\mathbf{Q}, \mathbf{R}, \mathbf{X}) \in \mathcal{QRX}$, which is defined by the following constraints:

$$\sum_{m \in M_{ps}, s \in S} R_{pms} \leq 1 \qquad \forall p \in P_{VIS} \qquad (d1')$$

$$\sum_{m \in M_{ps}, s \in S} \left( R_{pms} + Q_{pms} \right) \leq 1 \qquad \forall p \in P_{FLEX} \qquad (d2')$$

$$\sum_{g \in G_{pm}} X_{pmsg} \leq time_{pm} R_{pms} \qquad \forall p \in P_{VIS+}, m \in M_{ps}, s \in S \qquad (d3')$$

$$X_{pmsg} \geq trans_p R_{pms} \qquad \forall p \in P_{VIS+}, m \in M_{ps}, s \in S, g \in G_{pm} \qquad (d4')$$

$$\sum_{\substack{p \in P_{VIS+}, m \in M_p \\ s \in S | g \in G_{pm}}} R_{pms} + \sum_{\substack{p \in P_{SECRET+}, m \in M_p \\ s \in S | g \in G_{pm}}} Q_{pms} \leq \bar{n}_{PLATS} \qquad \forall g \in G \qquad (d5')$$

$$R_{p''m''s''} \leq \sum_{\substack{(p', p'') \in PP_{TETH}, \\ m' \in M_{p's'}, s' \in S | \\ range_{m'm''} \leq \\ teth\_range_{p''}}} R_{p'm's'} \qquad \forall p'' \in P_{TETH}, m \in M_{p''s''}, s'' \in S \qquad (d6')$$

$$R_{pms} \in \{0, 1\} \qquad \forall p \in P_{VIS+}, m \in M_{ps}, s \in S \qquad (d7')$$

$$X_{pmsg} \geq 0 \qquad \forall p \in P_{VIS+}, m \in M_{ps}, s \in S, g \in G \qquad (d8')$$

$$0 \leq Q_{pms} \leq 1 \qquad \forall p \in P_{SECRET+}, m \in M_{ps}, s \in S \qquad (d9')$$

These constraints are essentially the same as in **SCREEN**, with the exception of constraints (*d2′*), which require that each flexible platform selects either a single, active mission, or a mixed strategy of passive missions.

### 6. Limits on Defender's Secret Platforms

The inner maximization problem has feasible region $(\mathbf{Q},\mathbf{W}) \in \mathcal{QW}$, which is defined by the following constraints:

$$\sum_{m \in M_{ps}, s \in S} Q_{pms} \le 1 \qquad \forall p \in P_{SECRET} \qquad (d10')$$

$$\sum_{g \in G_{pm}} W_{pmsg} \le time_{pm} Q_{pms} \qquad \forall p \in P_{SECRET+}, m \in M_{ps}, s \in S \qquad (d11')$$

$$W_{pmsg} \ge trans_p Q_{pms} \qquad \forall p \in P_{SECRET+}, m \in M_{ps}, s \in S, g \in G_{pm} \qquad (d12')$$

$$0 \le Q_{pms} \le 1 \qquad \forall p \in P_{SECRET+}, m \in M_{ps}, s \in S \qquad (d13')$$

$$W_{pmsg} \ge 0 \qquad \forall p \in P_{SECRET+}, m \in M_{ps}, s \in S, g \in G \qquad (d14')$$

Constraints (*d10′*) through (*d14′*) require the same of the secret platforms as their visible counterparts, with the exception that **Q** is now continuous.

### 7. Conversion of SSCREEN to a MIP, SSCREEN$_{\mathbf{MIP}}$

Because the inner min-max model in **SSCREEN** forms a TPZSG with simultaneous play, the order of optimization can be reversed, so that the new problem becomes a max-max/min problem, as shown below.

**SSCREEN**

$$\max_{\substack{(\mathbf{Q},\mathbf{R},\mathbf{X}) \in \mathcal{QRX},\\ (\mathbf{Q},\mathbf{W}) \in \mathcal{QW}}} \left[ \begin{array}{lll} \min\limits_{\mathbf{Y}} \sum\limits_{(g,g') \in A} f_{g'}(\mathbf{W},\mathbf{X}) Y_{gg'} & & (a0') \\ \text{s.t.} \sum\limits_{g' | (g,g') \in A} Y_{gg'} - \sum\limits_{g' | (g',g) \in A} Y_{g'g} = \left\{ \begin{array}{lll} +subs & g = g^+ & [\alpha_{g^+}] \\ 0 & g \in G - \{g^+, g^-\} & [\alpha_g] \\ -subs & g = g^- & [\alpha_{g^-}] \end{array} \right\} & & (a1) \\ 0 \le Y_{ij} \le u & [\beta_{gg'}] & (a2) \end{array} \right]$$

We can temporarily fix **R**, **X**, **Q**, and **W**, take the dual of the inner, minimization problem, and then release **R**, **X**, **Q**, and **W**. This results in a maximizing MIP, **SSCREEN$_{\mathbf{MIP}}$**.

**SSCREEN$_{\mathbf{MIP}}$**

$$\max_{\substack{\mathbf{R},\mathbf{X},\mathbf{Q},\\ \mathbf{W},\boldsymbol{\alpha},\boldsymbol{\beta}}} n_{SUBS}\,\alpha_{g^+} - n_{SUBS}\,\alpha_{g^-} - u \sum_{(g,g')\in A} \beta_{gg'} \qquad (b0)$$

$$\text{s.t.} \quad \alpha_g - \alpha_{g'} - u\beta_{gg'} \le f_{g'}(\mathbf{W},\mathbf{X}) \qquad \forall (g,g')\in A \quad (b1')$$

$$\alpha_{g^+} = 0 \qquad (b2)$$

$$\beta_{gg'} \ge 0 \qquad \forall (g,g')\in A \qquad (b3)$$

$$(\mathbf{Q},\mathbf{R},\mathbf{X}) \in \mathcal{QRX}$$

$$(\mathbf{Q},\mathbf{W}) \in \mathcal{QW}$$

where $f_{g'}(\mathbf{W},\mathbf{X})$ is defined by (*a3*).

Given an optimal defense plan $(\mathbf{R}^*,\mathbf{X}^*)$ from **SSCREEN$_{\mathbf{MIP}}$**, we can recover an optimal attacker solution by fixing the first-stage variables $(\mathbf{R}^*,\mathbf{X}^*)$ in **SSCREEN$_{\mathbf{MIP}}$** and solving the resulting linear program. The resulting $(\mathbf{Q}^*,\mathbf{W}^*)$ are an optimal mixed strategy for the defender's secret platforms, and the dual variables to constraints (*b1′*) represent a corresponding optimal mixed strategy for the attacker.

In the case where the defender possesses only visible platforms, i.e., $P_{SECRET} = \varnothing$ and $P_{FLEX} = \varnothing$, then

$$f_{g'}(\mathbf{W},\mathbf{X}) = battery + \sum_{\substack{p\in P_{VIS}, m\in M_{ps}\\ s\in S | g'\in G_{pm}}} r'_{psg'} X_{pmsg'}$$

and (*a0′*) reduces to (*a0*), (*b1′*) reduces to (*b1*), and $\mathcal{QRX}$ reduces to $\mathcal{RX}$. Therefore, **SSCREEN** and **SCREEN** are identical when the defender has only visible platforms.

## 8. Decomposition of SSCREEN$_{\mathbf{MIP}}$

**SSCREEN$_{\mathbf{MIP}}$** can be difficult to solve using brute-force MIP techniques. Therefore, we decompose the optimization as follows.

**New Indices**

| | |
|---|---|
| $k \in K$ | decomposition iteration |

**New Data**

| | |
|---|---|
| $\hat{\mathbf{Q}}^k, \hat{\mathbf{R}}^k, \hat{\mathbf{W}}^k, \hat{\mathbf{X}}^k$ | defender's plans for iteration $k$ |
| $\hat{\mathbf{Y}}^k$ | attacker's plans for iteration $k$ |
| $\hat{\mathcal{Y}}$ | set containing attacker plans $\left\{\hat{\mathbf{Y}}^1, ..., \hat{\mathbf{Y}}^K\right\}$ |

We define the attacker's subproblem $\mathbf{AMIN}(\hat{\mathbf{W}}, \hat{\mathbf{X}})$ as

$\mathbf{AMIN}(\hat{\mathbf{W}}, \hat{\mathbf{X}})$

$$Z_{\min}(\hat{\mathbf{W}}, \hat{\mathbf{X}}) = \min_{\mathbf{Y}} \sum_{(g,g') \in A} f_{g'}(\hat{\mathbf{W}}^k, \hat{\mathbf{X}}^k) Y_{gg'} \qquad (s0)$$

$$\text{s.t.} \quad \sum_{g' | (g,g') \in A} Y_{gg'} - \sum_{g' | (g',g) \in A} Y_{g'g} = \left\{ \begin{array}{lll} +n_{SUBS} & g = g^+ & [\alpha_{g^+}] \\ 0 & g \in G - \{g^+, g^-\} & [\alpha_g] \\ -n_{SUBS} & g = g^- & [\alpha_{g^-}] \end{array} \right\} \quad (s1)$$

$$0 \le Y_{ij} \le u \qquad [\beta_{gg'}] \quad (s2)$$

where

$$f_{g'}(\hat{\mathbf{W}}^k, \hat{\mathbf{X}}^k) \equiv battery + \sum_{\substack{p \in P_{VIS}, m \in M_{ps} \\ s \in S | g' \in G_{pm}}} r'_{psg'} \hat{X}^k_{pmsg'} + \sum_{\substack{p \in P_{FLEX}, \\ m \in M_{ps} | g' \in G_{pm}}} r'_{p,s_1,g'} \hat{X}^k_{pm,s_1,g'} + \sum_{\substack{p \in P_{FLEX}, \\ m \in M_{ps} | g' \in G_{pm}}} r'_{p,s_2,g'} \hat{W}^k_{pm,s_2,g'} + \sum_{\substack{p \in P_{SECRET}, m \in M_{ps} \\ s \in S | g' \in G_{pm}}} r'_{psg'} \hat{W}^k_{pmsg'}$$

We define the defender's master problem $\mathbf{DMAX}(\hat{\mathcal{Y}})$ as

$$\mathbf{DMAX}(\hat{\mathcal{Y}})$$

$$Z_{\max}\left(\hat{\mathcal{Y}}\right)=\max_{\substack{\mathbf{R},\mathbf{X},\\ \mathbf{Q},\mathbf{W}}} Z \qquad (m0)$$

$$\text{s.t.} \quad Z \le \sum_{(g,g')\in A} f_{g'}(\mathbf{W},\mathbf{X})\hat{Y}^k_{gg'} \qquad \forall \hat{\mathbf{Y}}^k \in \hat{\mathcal{Y}} \qquad (m1)$$

$$(\mathbf{Q},\mathbf{R},\mathbf{X}) \in \mathcal{QRX}$$

$$(\mathbf{Q},\mathbf{W}) \in \mathcal{QW}$$

The complete decomposition algorithm follows:

**Algorithm MAXMIN**

Input: Data for defender's problem, optimality tolerance $\varepsilon \ge 0$;

Output: $\varepsilon$-optimal **SSCREEN** defense plan $(\mathbf{Q}^*,\mathbf{R}^*,\mathbf{W}^*,\mathbf{X}^*)$, and corresponding attacker course of action (COA) $\mathbf{Y}^*$;

1. Initialize best lower bound $Z_{LB} \leftarrow -\infty$, best upper bound $Z_{UB} \leftarrow \infty$, define the incumbent, null **SSCREEN** defense plan $(\hat{\mathbf{W}}^1,\hat{\mathbf{X}}^1) \leftarrow (\mathbf{0},\mathbf{0})$ as the best found so far, set $\hat{\mathcal{Y}} \leftarrow \varnothing$, and set iteration counter $K \leftarrow 1$;

2. **Subproblem**: Solve subproblem $\mathbf{AMIN}(\hat{\mathbf{W}},\hat{\mathbf{X}})$ to determine the optimal attacker's COA $\hat{\mathbf{Y}}^K$ given defense plan $(\hat{\mathbf{W}}^K,\hat{\mathbf{X}}^K)$; the bound on the associated objective is $Z_{\min}(\hat{\mathbf{W}}^K,\hat{\mathbf{X}}^K)$;

3. If $Z_{LB} < Z_{\min}(\hat{\mathbf{W}}^K,\hat{\mathbf{X}}^K)$, set $Z_{LB} \leftarrow Z_{\min}(\hat{\mathbf{W}}^K,\hat{\mathbf{X}}^K)$ and record improved incumbent **SSCREEN** defense plan $(\mathbf{W}^*,\mathbf{X}^*) \leftarrow (\hat{\mathbf{W}}^K,\hat{\mathbf{X}}^K)$;

4. If ( $Z_{UB} - Z_{LB} \le \varepsilon$ ) go to **End**;

5. **Master Problem:** Set $\hat{\mathcal{Y}} \leftarrow \hat{\mathcal{Y}} \cup \left\{\hat{\mathbf{Y}}^K\right\}$ and solve master problem **DMAX**($\hat{\mathcal{Y}}$) to determine an optimal defense plan $\left(\hat{\mathbf{W}}^{K+1}, \hat{\mathbf{X}}^{K+1}\right)$. The bound on the objective is $Z_{\max}(\hat{\mathcal{Y}})$;

6. If $Z_{UB} > Z_{\max}(\hat{\mathcal{Y}})$ set $Z_{UB} \leftarrow Z_{\max}(\hat{\mathcal{Y}})$;

7. If $(Z_{UB} - Z_{LB} \le \varepsilon)$ go to **End**;

8. Set $K \leftarrow K+1$ and go to step (2) (**Subproblem)**;

9. **End**: Solve **SSCREEN$_{\mathbf{MIP}}$** with fixed defense plan $(\mathbf{W}^*, \mathbf{X}^*)$ to recover $(\mathbf{Q}^*, \mathbf{R}^*)$. Fix $(\mathbf{R}^*, \mathbf{X}^*)$ and again solve **SSCREEN$_{\mathbf{MIP}}$** as a linear program; the dual variables to constraints (*b1′*) correspond to the attacker's optimal mixed strategy $\mathbf{Y}^*$. Print "$(\mathbf{Q}^*, \mathbf{R}^*, \mathbf{W}^*, \mathbf{X}^*)$ is an $\varepsilon$-optimal **SSCREEN** defense plan, and $\mathbf{Y}^*$ is the attacker's response to that plan," and halt.

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# III. RESULTS AND ANALYSIS

This chapter demonstrates the capabilities of G-TAMP (**SSCREEN**, the D-A/D model) by applying the model to a set of realistic, although simple, scenarios. All 4W grid geography and numerical values are hypothetical; the reader can reproduce any of our experiments from the data in Appendix B.

For all scenarios, we assume the following: (1) the active sensor mode always provides better performance than the passive sensor mode in each cell; (2) each platform has an assigned time of four hours on station; (3) each candidate mission consists of one, two, or three contiguous cells, and for each platform, we enumerate all possible missions; (4) all grid cells measure 5 nautical miles (nm) by 5 nm, or 25 nm$^2$ in area; (5) when a helicopter is included in the scenario, it is tethered to its corresponding surface platform (e.g., Helo1 is tethered to Surf1) with a tether range of 10 nm; (6) the attacker employs a single SSK; (7) for probability of detection $(P^D)$ calculation, the SSK transits a searched cell in 1.5 hours at 4 knots. These modeling assumptions can be modified easily, at the discretion of a planner.

Using the General Algebraic Modeling System (GAMS 2008) with Cplex 11 (ILOG 2007), a desktop personal computer with an Intel 3.73 GHz processor solves four of the five examples exactly in less than ten minutes; however, one example requires 13 hours to solve to within a relative optimality gap of 5%. The largest problem has 14,727 constraints, 9,105 continuous variables, and 3,288 binary variables.

## A. EXAMPLE ONE: BASIC SCENARIO

The first scenario illustrates how **SSCREEN** recommends an intuitively appealing battle group screen to protect an HVU. It makes use of the geography in Figure 6.

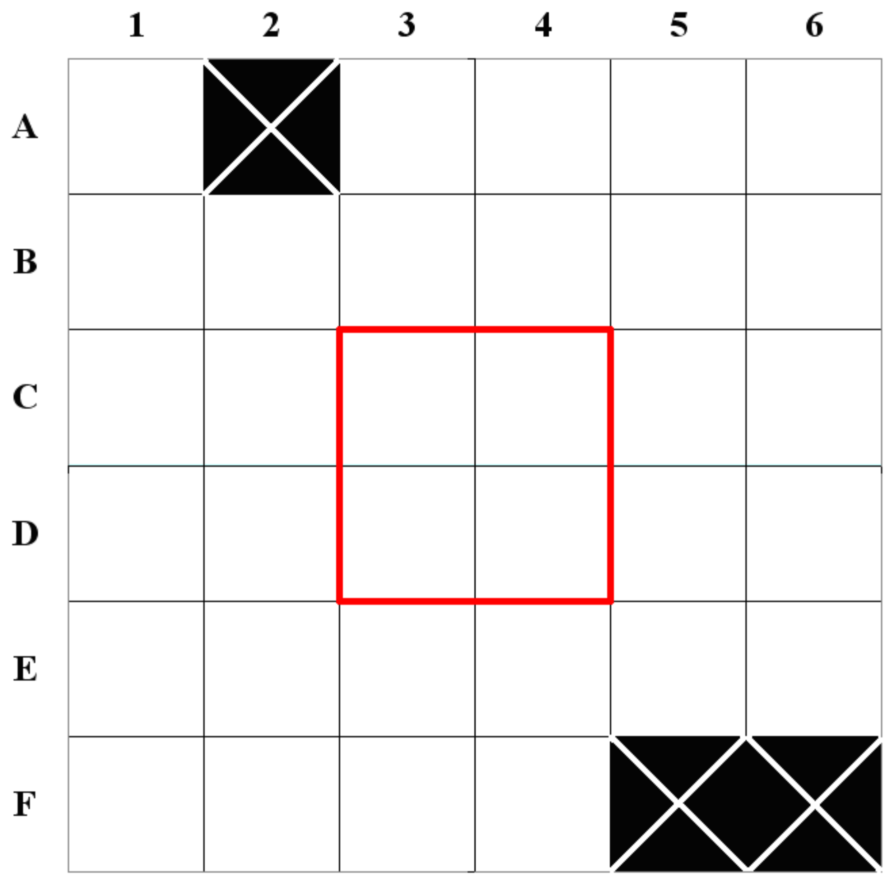


**Figure 6. Basic scenario, 4W grid geography**

For this scenario, cells C3, C4, D3, and D4 (boxed) comprise the region of protected cells; cells A2, F5, and F6 are impassable.

We assume the ocean to be non-homogeneous; Figure 7 depicts the coverage rate in each cell.

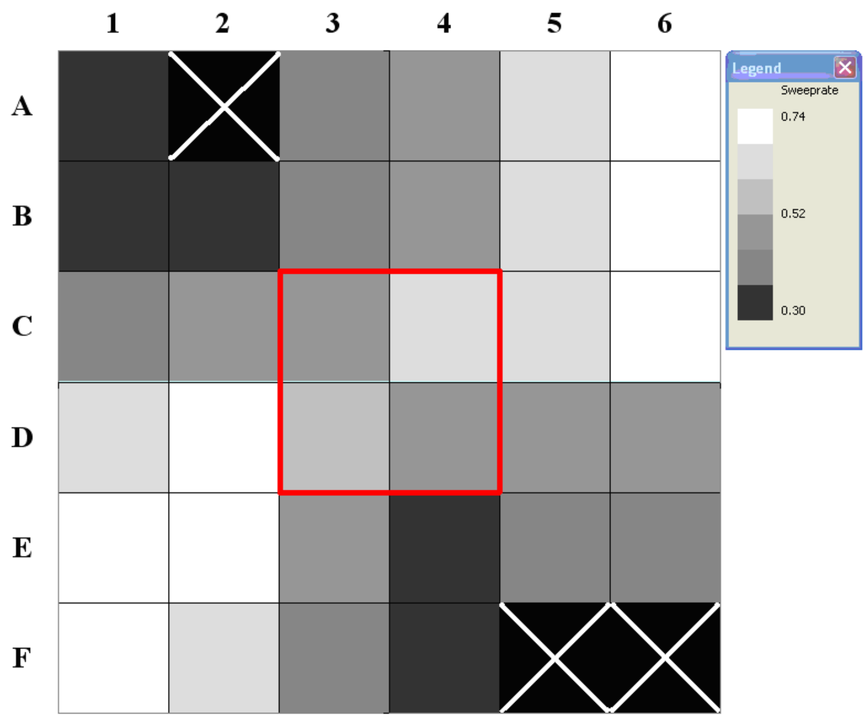


**Figure 7. Basic scenario, coverage rates**
Friendly platforms can search light-colored cells more easily than dark-colored cells.

To protect the HVU, friendly forces employ the platforms listed in Table 2.

| Platform | Type |
|---|---|
| Surf1 | Visible |
| Surf2 | Visible |
| Surf3 | Visible |
| Helo1 | Visible |
| P31 | Visible |
| SSN1 | Flexible |

**Table 2. Basic scenario, available platforms**

An optimal solution positions platforms as shown in Figure 8; Table 3 shows the optimal corresponding mission details. Because the screen of defending platforms is only one cell wide, the objective value of 1.39 means that the attacker sends an SSK through exactly one cell that is searched 1.39 times by defending platforms over the mission's time horizon (four hours). This objective value corresponds to a conservatively computed $P^D$ of 0.41; therefore, the attacker's single SSK will be detected with this probability (see Appendix A).

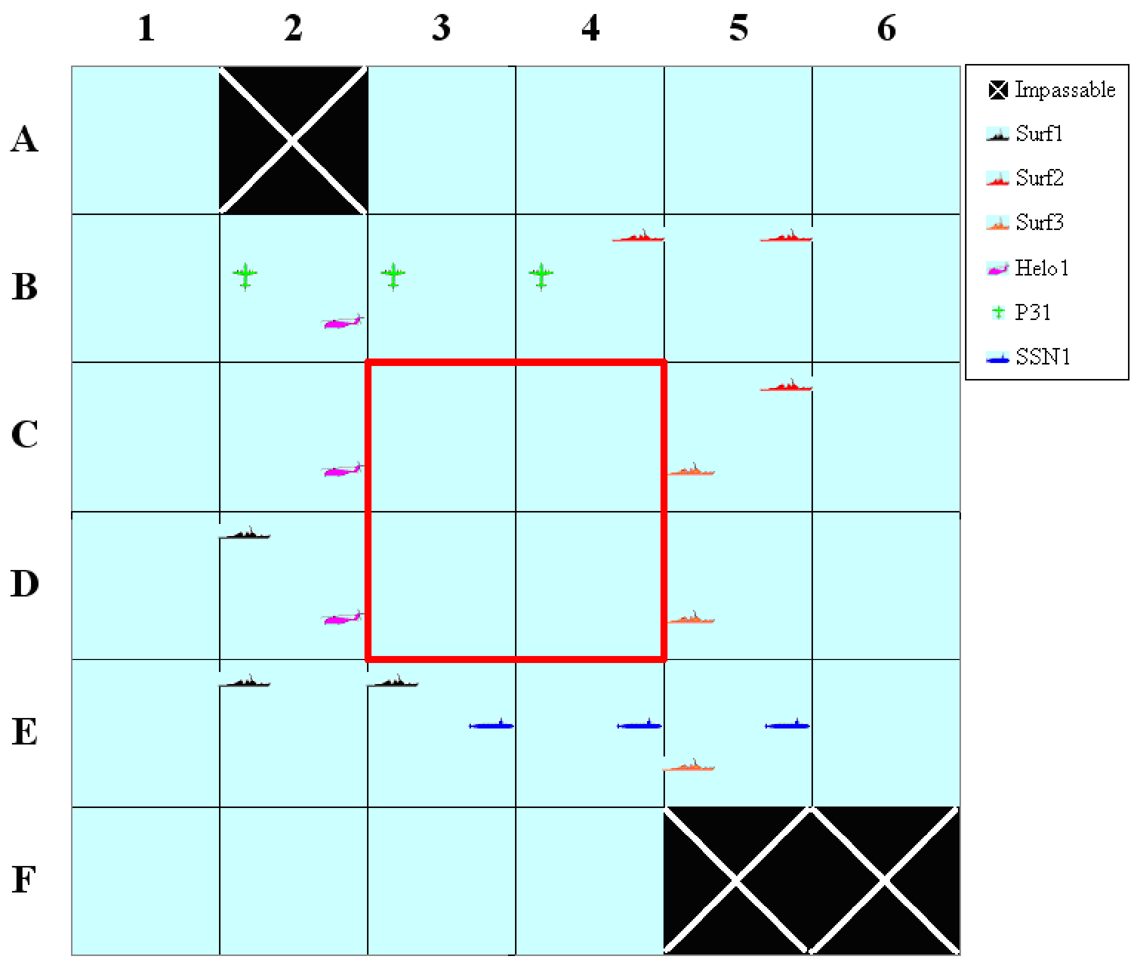


**Figure 8. Basic scenario, defensive platform positions**
In this scenario, all defenders choose active sensor modes; they form a tight screen around the HVU. Each defensive platform has a deterministic mission assignment to patrol a given set of cells.

| Surf1 | active | Surf2 | active | Surf3 | active | Helo1 | active | P31 | active | SSN1 | active |
|---|---|---|---|---|---|---|---|---|---|---|---|
| Cell | Time | Cell | Time | Cell | Time | Cell | Time | Cell | Time | Cell | Time |
| D2 | 1.09 | B4 | 1.23 | C5 | 0.65 | B2 | 1.60 | B2 | 1.07 | E3 | 0.70 |
| E2 | 1.46 | B5 | 1.75 | D5 | 2.27 | C2 | 2.06 | B3 | 2.16 | E4 | 2.13 |
| E3 | 1.45 | C5 | 1.03 | E5 | 1.08 | D2 | 0.33 | B4 | 0.77 | E5 | 1.17 |

**Table 3. Basic scenario, defensive platform mission assignments**
A mission tells each platform where to search, for how long, and which sensor mode to use. Here, each platform chooses to patrol three cells for the times shown (in hours). SSN1 chooses to use active sonar for increased sensor performance.

In this scenario, the defenders form a tight screen around the HVU so that the pressure in any cell adjacent to a protected cell is 1.39. Therefore, no matter which path the attacker chooses, he encounters the same level of search effort and the same probability of detection. Note that SSN1 chooses the active sensor mode, and the attacker therefore observes its location.

## B. EXAMPLE TWO: SHORT-HANDED SCENARIO

The second scenario demonstrates a simple case in which a defensive platform chooses to remain secret. It makes use of the basic scenario geography and ocean environment shown in Figures 6 and 7, respectively.

Here, the defender has only a single surface ship and submarine to protect the HVU (see Table 4).

| **Platform** | **Type** |
|---|---|
| Surf1 | Visible |
| SSN1 | Flexible |

**Table 4. Short-handed scenario, available platforms**

The optimal solution positions platforms as shown in Figure 9; Table 5 summarizes the search plan. The solution provides an objective value of 0.33, or an approximate $P^D$ of 0.12.

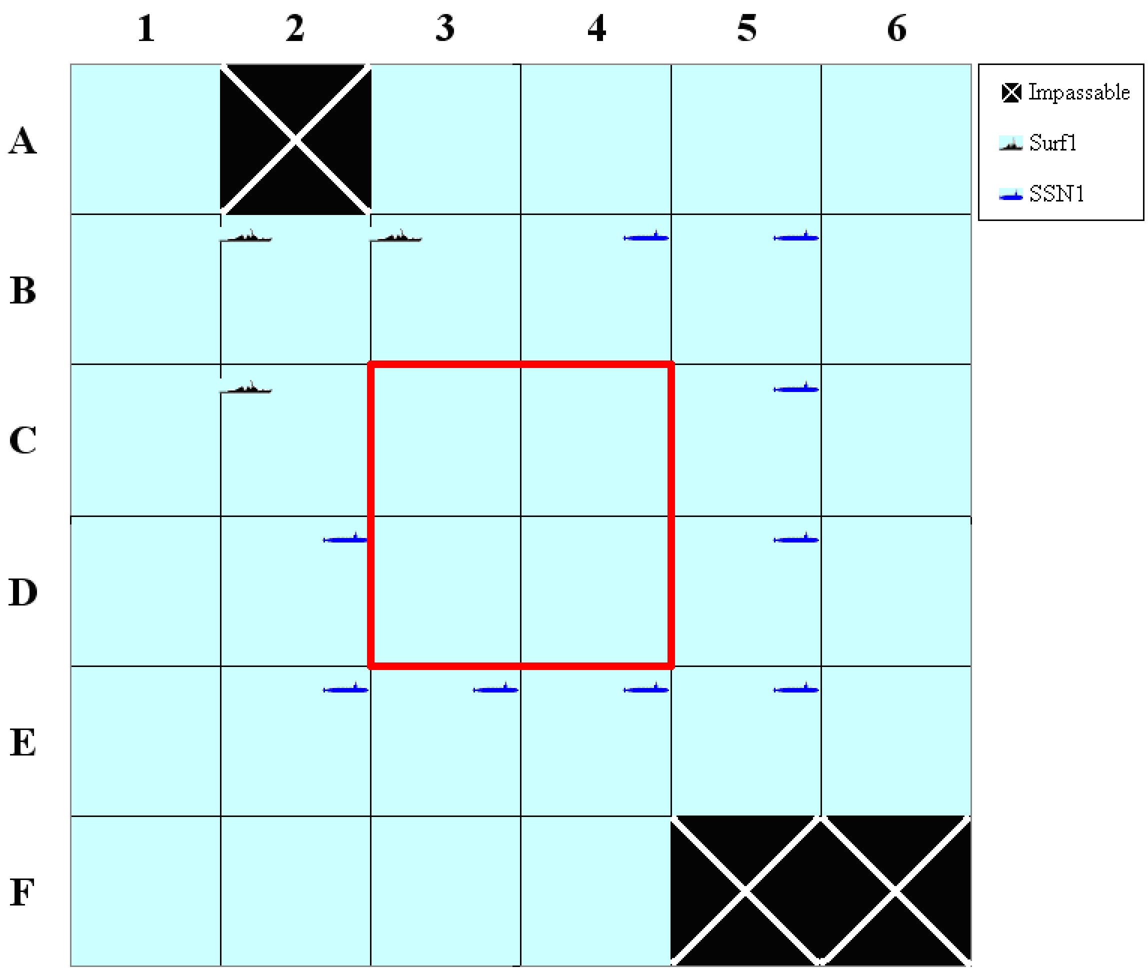


**Figure 9. Short-handed scenario, defensive platform positions**

Because two platforms cannot patrol the entire perimeter of the protected region, SSN1 remains secret, and chooses a mixed strategy to randomize its actions. Each cell with a submarine icon potentially receives some pressure from SSN1.

| Surf1 | active |
|---|---|
| **Cell** | **Time** |
| B2 | 0.76 |
| B3 | 0.65 |
| C2 | 2.59 |

| SSN1 | passive | | |
|---|---|---|---|
| **Prob** | **0.078** | **Prob** | **0.092** |
| B4 | 4.00 | D2 | 3.30 |
| **Prob** | **0.114** | E2 | 0.70 |
| D5 | 4.00 | **Prob** | **0.080** |
| **Prob** | **0.079** | B4 | 0.70 |
| E3 | 4.00 | B5 | 3.30 |
| **Prob** | **0.148** | **Prob** | **0.014** |
| E5 | 4.00 | E2 | 3.30 |
| **Prob** | **0.266** | E3 | 0.70 |
| E2 | 0.70 | **Prob** | **0.130** |
| E3 | 0.70 | B4 | 0.70 |
| E4 | 2.60 | B5 | 0.70 |
| | | C5 | 2.60 |

**Table 5. Short-handed scenario, defensive platform mission assignments**
Surf1, a visible platform, chooses a single mission as shown. SSN1, a flexible platform, chooses to remain secret and therefore uses a mixed strategy that randomizes its actions. The table on the right shows the probability with which SSN1 should perform each mission.

In this case, because we limit mission size to three cells, two platforms cannot possibly patrol the entire perimeter of the protected region. If SSN1 chooses an visible mission, the attacker can observe the locations of both defenders, and take an unopposed route to the HVU. Therefore, SSN1 remains secret, and randomizes its patrol plan to remain unpredictable to the attacker's SSK(s). The attacker likewise randomizes his actions to keep the defender guessing. Figure 10 shows an optimal attacker course of action (COA) and the cell coverage rates for comparison.

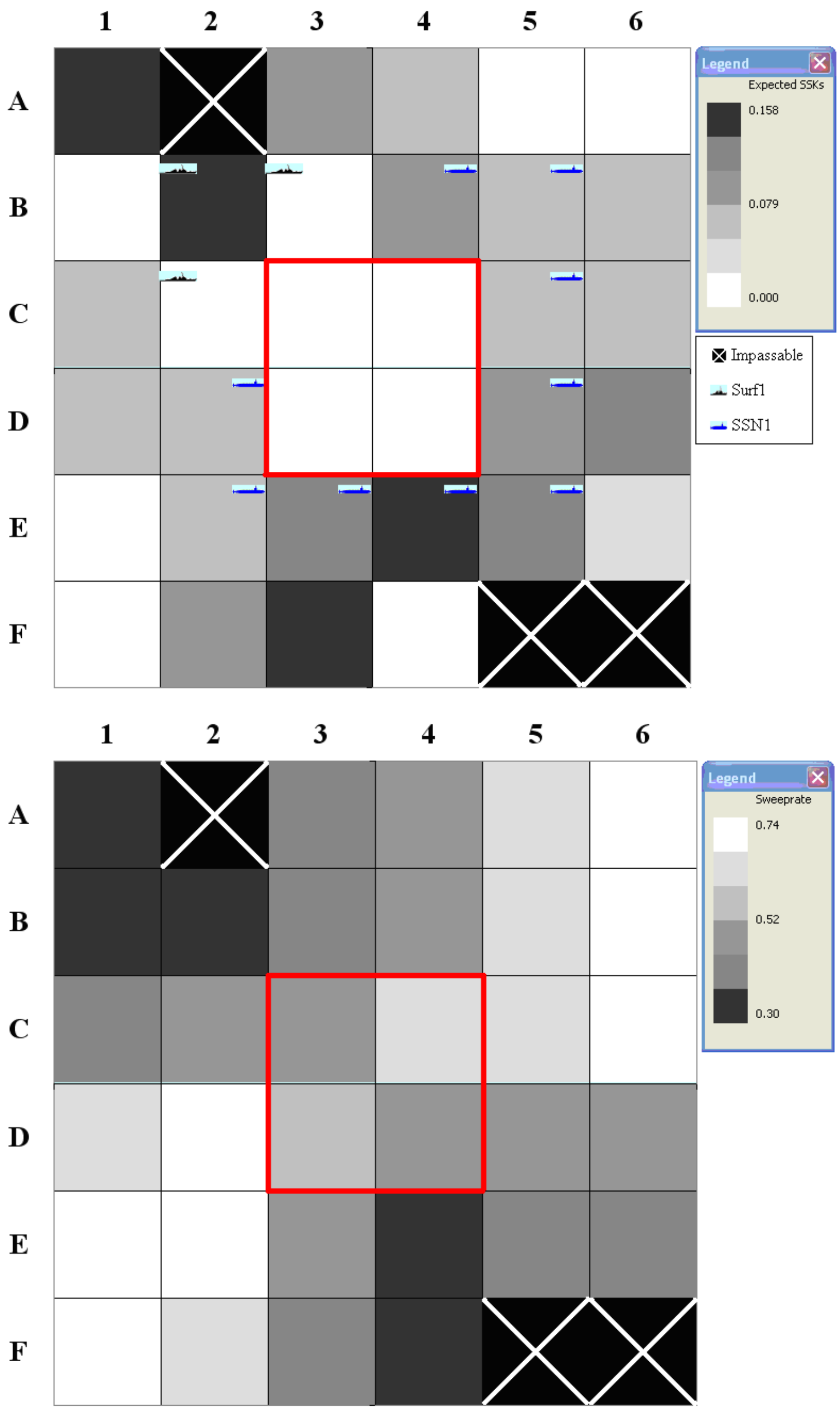


**Figure 10. Short-handed scenario, attacker COA and coverage rates (top and bottom, respectively)**

In the top figure, dark cells show the attacker's preferred routes; the attacker avoids light cells. The coverage rates are shown bottom for comparison. Note how dark cells here (which are hard to search) correspond to dark cells above (the likely SSK routes).

We see that the attacker prefers to traverse cells with low coverage rates, despite the fact that each cell on the perimeter of the protected region has the same (expected value of) pressure of 0.33, or an approximate $P^D$ of 0.12. If the attacker chooses to transit an easily searchable cell, and the submarine chooses the same cell, the attacker accumulates a high pressure cost and will likely be detected. Likewise, even though SSN1 spends a majority of its time patrolling hard-to-search cells, the attacker knows that he will be more difficult to detect, so prefers these paths.

## C. EXAMPLE THREE: OCEAN-INFLUENCE SCENARIO

The third scenario demonstrates how a non-homogeneous ocean environment creates a non-intuitive solution. We again use the basic scenario geography of Figure 6, with the coverage rates shown in Figure 11. The protected cells lie in a region of hard-to-search cells resulting from, say, an ocean front or current. The defender has the platforms listed in Table 6 with which to search.

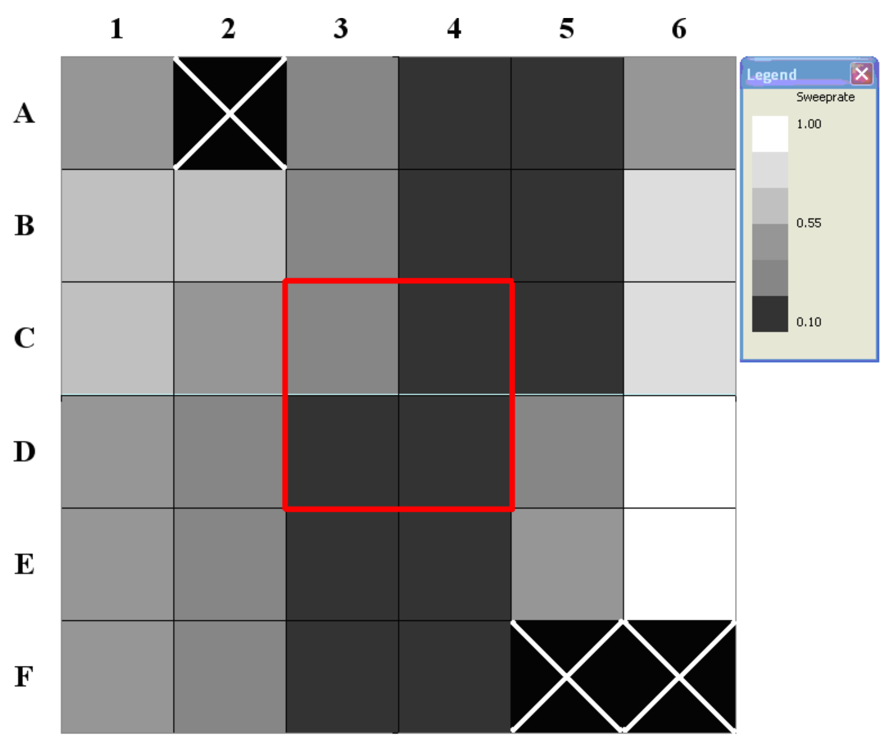


**Figure 11. Ocean-influence scenario, coverage rates**

The protected region is surrounded by cells with low coverage rates, as would be the case if the HVU took station near an ocean current (e.g., the Gulf Stream or Kuroshio) or near an ocean front (for background on the ocean environment, see Pickard and Emery 1990).

| Platform | Type |
|---|---|
| Surf1 | Visible |
| Surf2 | Visible |
| Surf3 | Visible |
| Helo1 | Visible |
| P31 | Visible |
| SSN1 | Flexible |

**Table 6. Ocean-influence scenario, available platforms**

Figure 12 shows the optimal locations for each platform to search; Table 7 details the search plan, which provides an objective value of 0.67, or a $P^D$ of 0.22.

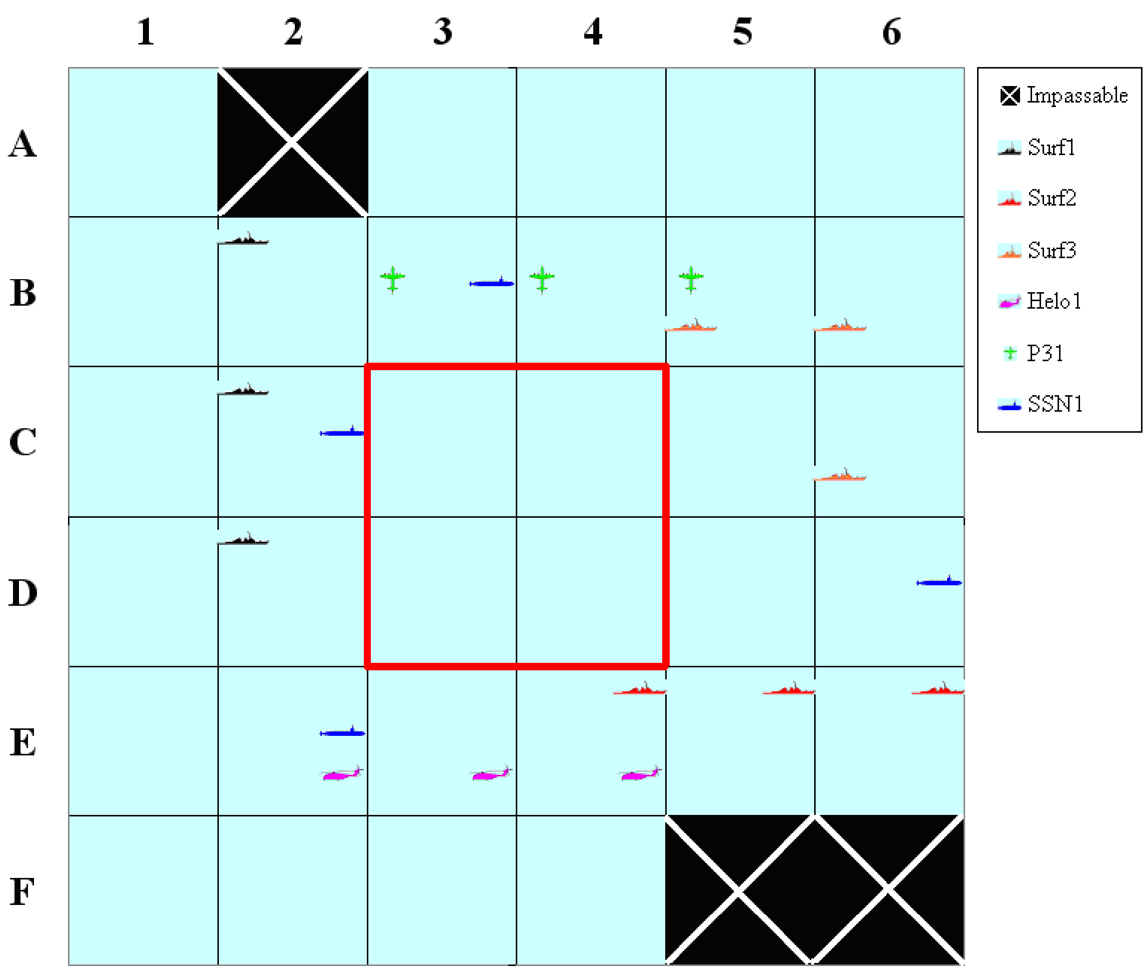


**Figure 12. Ocean-influence scenario, defensive platform positions**
The ocean current forces a non-obvious solution. Because cells C5 and D5 have low coverage rates, the screen moves outward to regions more easily searched. SSN1 chooses to operate in secret.

| Surf1 | active | Surf2 | active | Surf3 | active | Helo1 | active | P31 | active |
|---|---|---|---|---|---|---|---|---|---|
| **Cell** | **Time** | **Cell** | **Time** | **Cell** | **Time** | **Cell** | **Time** | **Cell** | **Time** |
| B2 | 0.96 | E4 | 2.40 | B5 | 2.61 | E2 | 0.47 | B3 | 0.30 |
| C2 | 1.05 | E5 | 1.07 | B6 | 0.70 | E3 | 2.67 | B4 | 3.05 |
| D2 | 2.00 | E6 | 0.53 | C6 | 0.70 | E4 | 0.87 | B5 | 0.65 |

| SSN1 | passive |
|---|---|
| **Prob** | **0.475** |
| B3 | 4.00 |
| **Prob** | **0.037** |
| C2 | 4.00 |
| **Prob** | **0.123** |
| D6 | 4.00 |
| **Prob** | **0.365** |
| E2 | 4.00 |

**Table 7. Ocean-influence scenario, defensive platform mission assignments**

SSN1 chooses to remain secret, and most often searches cells with low coverage rates.

The ocean environment leads to the screen expanding outward to regions more easily searched. SSN1 remains passive in order to capitalize on its secrecy: most of the time, it searches cells B3 and E2 to aid the other, active searchers, but it occasionally patrols D6, a cell with a high coverage rate, to deter the attacker.

Figure 13 shows the attacker's COA and the coverage rates for comparison. Once again, we see that the attacker prefers to transit through hard-to-search cells. The majority of his paths arrive from the top or bottom of the grid as he travels toward the HVU through the ocean current. Even though the attacker observes no defender in cell D6, he usually avoids it, fearing that a secret platform has taken station in water with good detection conditions.

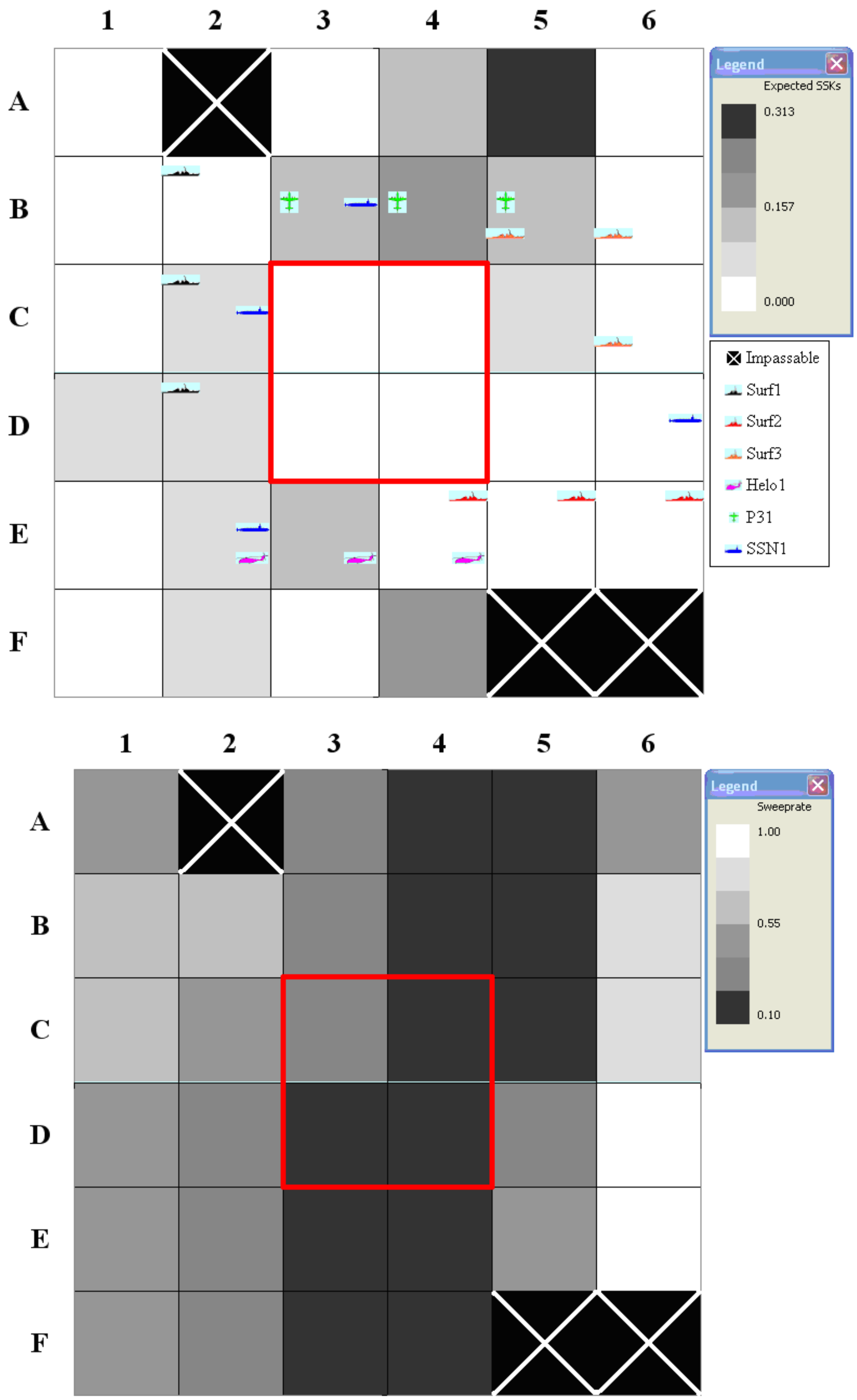


**Figure 13. Ocean-influence scenario, attacker COA and coverage rates (top and bottom, respectively)**

The attacker (top figure) prefers to route his SSK from the top and bottom of the grid, where it can take advantage of the poor search conditions (see bottom figure). He usually avoids the apparently empty D6.

## D. EXAMPLE FOUR: MULTIPLE-HVU SCENARIO

The fourth scenario demonstrates how planners can employ G-TAMP to optimally allocate ASW platforms among multiple aircraft carrier operating areas (CVOAs) within a single 4W grid. The scenario makes use of the geography in Figure 14; the coverage rates are shown in Figure 15. Table 8 shows the platforms that the defender must allocate among the two, geographically separated, protected cells. In addition to the tethers placed on Helo1 and Helo2, the defender prevents mutual interference between the submarines; SSN1 and SSN2 each have assigned admissible cells that the other cannot enter, as shown in Figure 16.

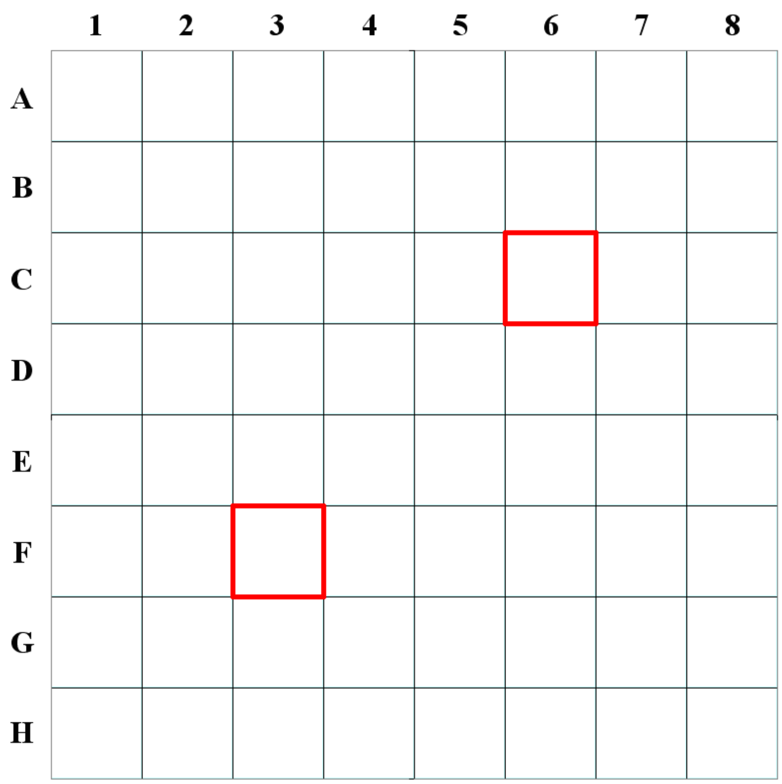


**Figure 14. Multiple-HVU scenario, 4W grid geography**
The defender stations one HVU in each of two geographically separated, protected cells (F3, C6), and must allocate defensive platforms among them.

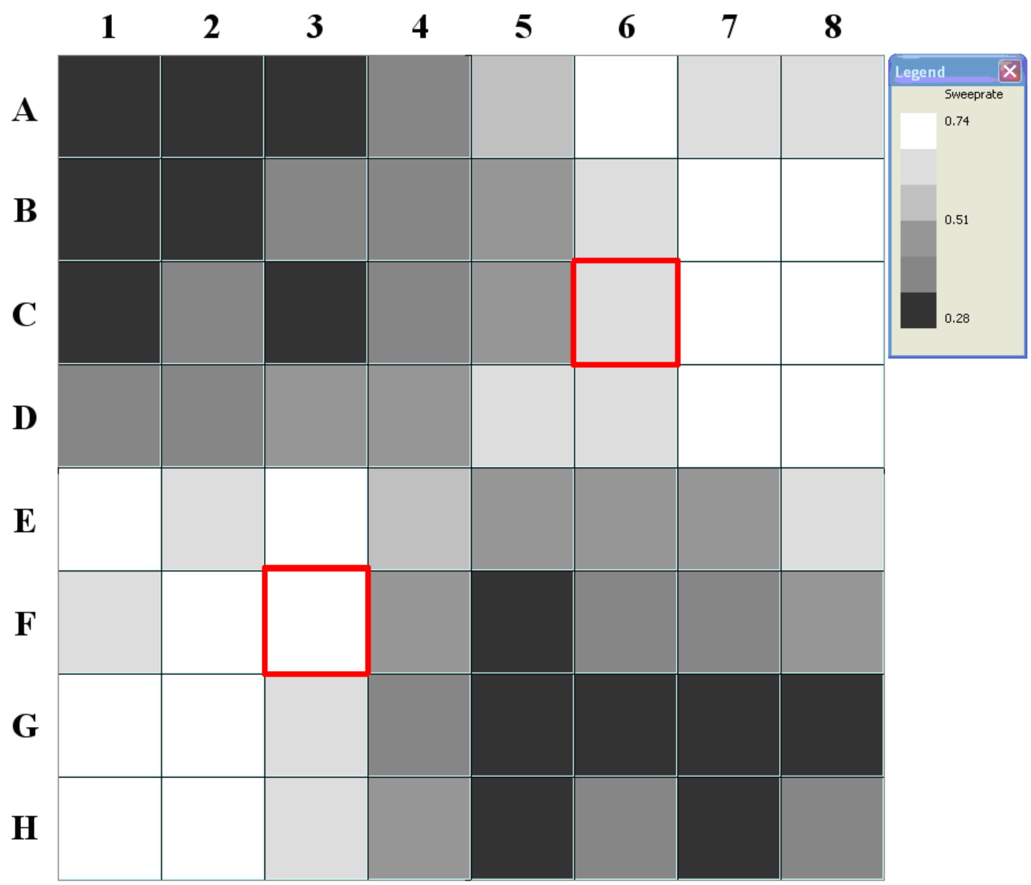


**Figure 15. Multiple-HVU scenario, coverage rates**

As before, the defender can easily search light-colored cells, while dark cells are harder to search. The defender wisely places his CVOAs in easily searched water to facilitate detection of approaching SSKs.

| Platform | Type |
|---|---|
| Surf1 | Visible |
| Surf2 | Visible |
| Surf3 | Visible |
| Helo1 | Visible |
| Helo2 | Visible |
| P31 | Visible |
| SSN1 | Flexible |
| SSN2 | Secret |

**Table 8. Multiple-HVU scenario, available platforms**

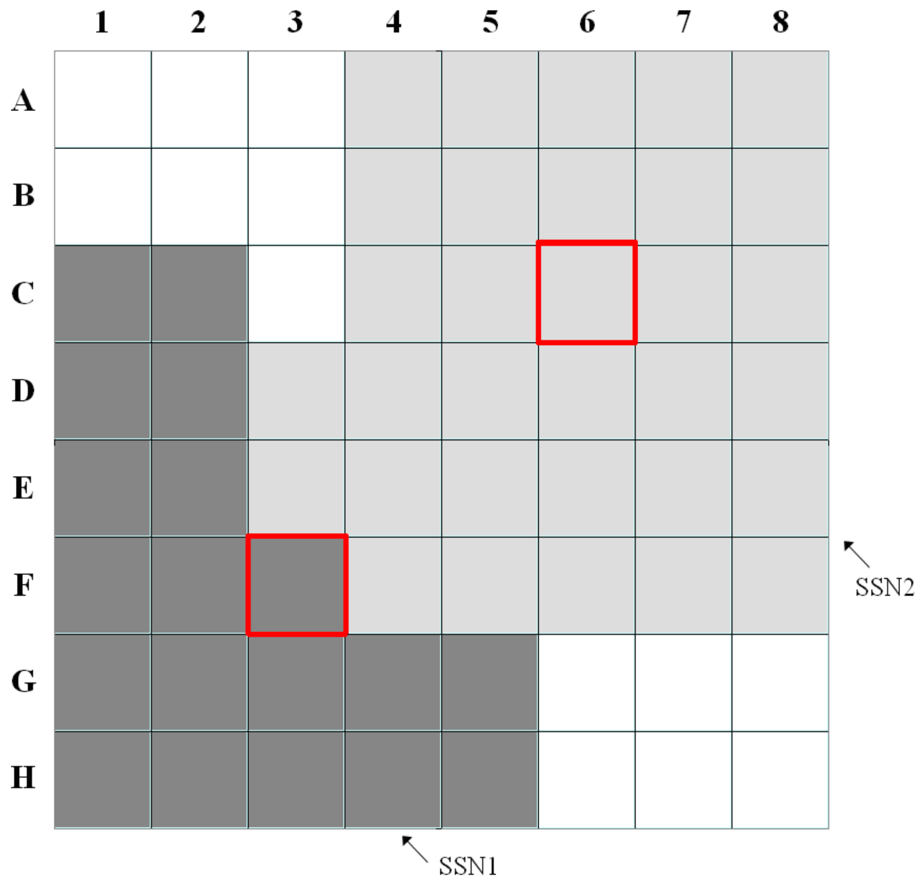


**Figure 16. Multiple-HVU scenario, allowed cells for SSNs**
In order to prevent interference between the submarines, SSN1 must remain in its assigned area (dark-shaded cells), while SSN2 must stay in the light-shaded cells.

Figure 17 shows the optimal placement of the platforms, and Table 9 shows the search plans, which provide an objective value of 1.76, or a $P^D$ of 0.48. SSN1, a flexible platform, chooses the active sensor mode. SSN2, forced to remain secret, chooses a mixed strategy of missions. Note that it is optimal to occasionally move SSN2 from one CVOA to the other, in order to keep the attacker guessing.

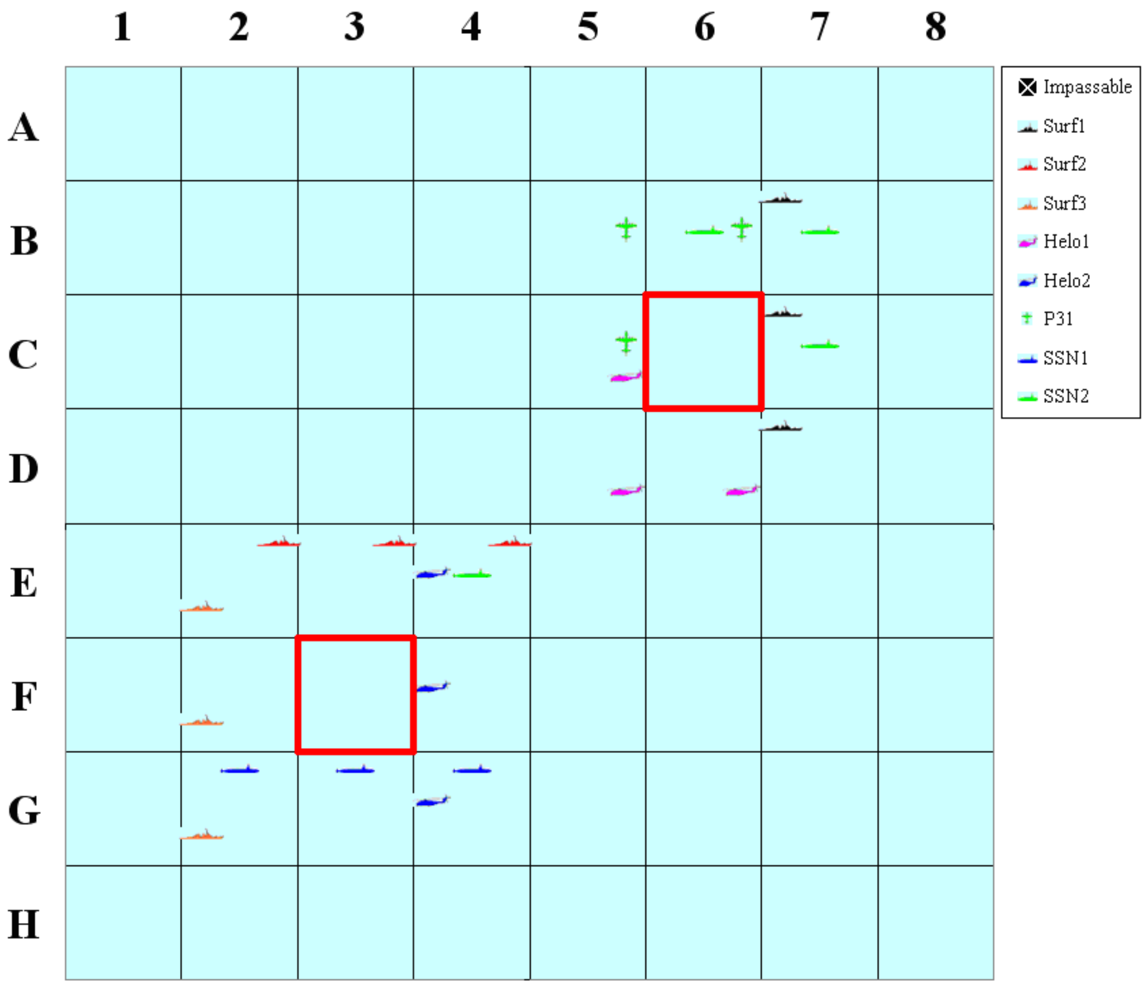


**Figure 17. Multiple-HVU scenario, defensive platform positions**
Four (three) visible platforms are uniquely assigned to the CVOA associated with cell F3 (C6). However, SSN2, a secret platform, performs a mission near F3 with probability 0.108, and a mission near C6 with probability 0.892 (see Table 9).

| Surf1 | active | Surf2 | active | Surf3 | active | Helo1 | active | Helo2 | active | P31 | active | SSN1 | active |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| **Cell** | **Time** | **Cell** | **Time** | **Cell** | **Time** | **Cell** | **Time** | **Cell** | **Time** | **Cell** | **Time** | **Cell** | **Time** |
| B7 | 1.51 | E2 | 1.01 | E2 | 1.12 | C5 | 0.41 | E4 | 0.80 | B5 | 2.13 | G2 | 0.70 |
| C7 | 0.50 | E3 | 1.89 | F2 | 1.94 | D5 | 1.88 | F4 | 2.60 | B6 | 0.12 | G3 | 1.43 |
| D7 | 1.99 | E4 | 1.10 | G2 | 0.94 | D6 | 1.72 | G4 | 0.60 | C5 | 1.75 | G4 | 1.87 |

| SSN2 | passive | | |
|---|---|---|---|
| **Prob** | **0.313** | **Prob** | **0.108** |
| C7 | 4.00 | E4 | 4.00 |
| **Prob** | **0.557** | **Prob** | **0.022** |
| B6 | 3.30 | B7 | 4.00 |
| B7 | 0.70 | | |

**Table 9. Multiple-HVU scenario, defensive platform mission assignments**

The defender distributes his platforms among CVOAs. SSN1 uses active sonar; SSN2 remains secret with a mixed strategy that divides its time between CVOAs.

## E. EXAMPLE FIVE: CHOKE-POINT SCENARIO

In the fifth and last scenario, we assume that the attacker sends an SSK from the west (left) side of the grid, and that he must transit through a navigational choke-point. The defender, privy to intelligence or cuing from another source, wishes to prevent the attacker's passage. We model this by placing the protected region on the right side of the 4W grid, as shown in Figure 18. The defender employs the platforms in Table 10, and the coverage rates are shown in Figure 19.

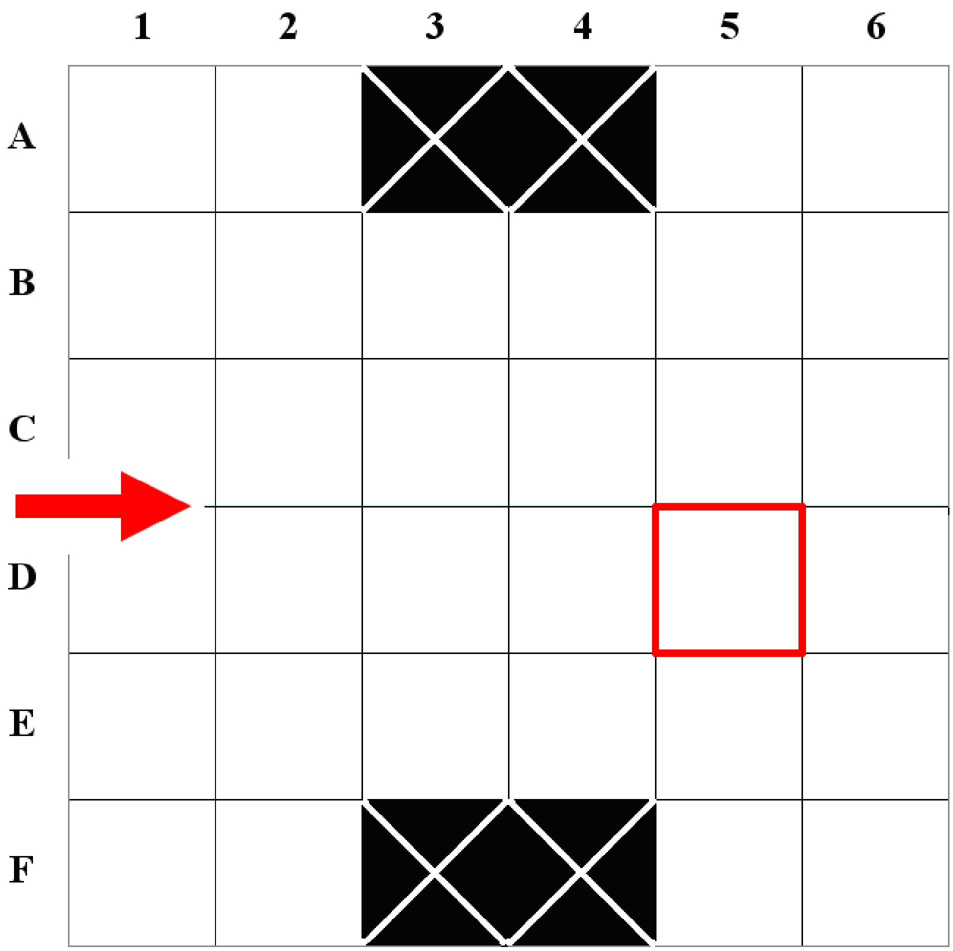


**Figure 18. Choke-point scenario, 4W grid geography**
The attacker sends an SSK through the choke-point from left to right. The defender desires to prevent his passage.

| Platform | Type |
|---|---|
| Surf1 | Visible |
| Helo1 | Visible |
| SSN1 | Flexible |

**Table 10. Choke-point scenario, available platforms**

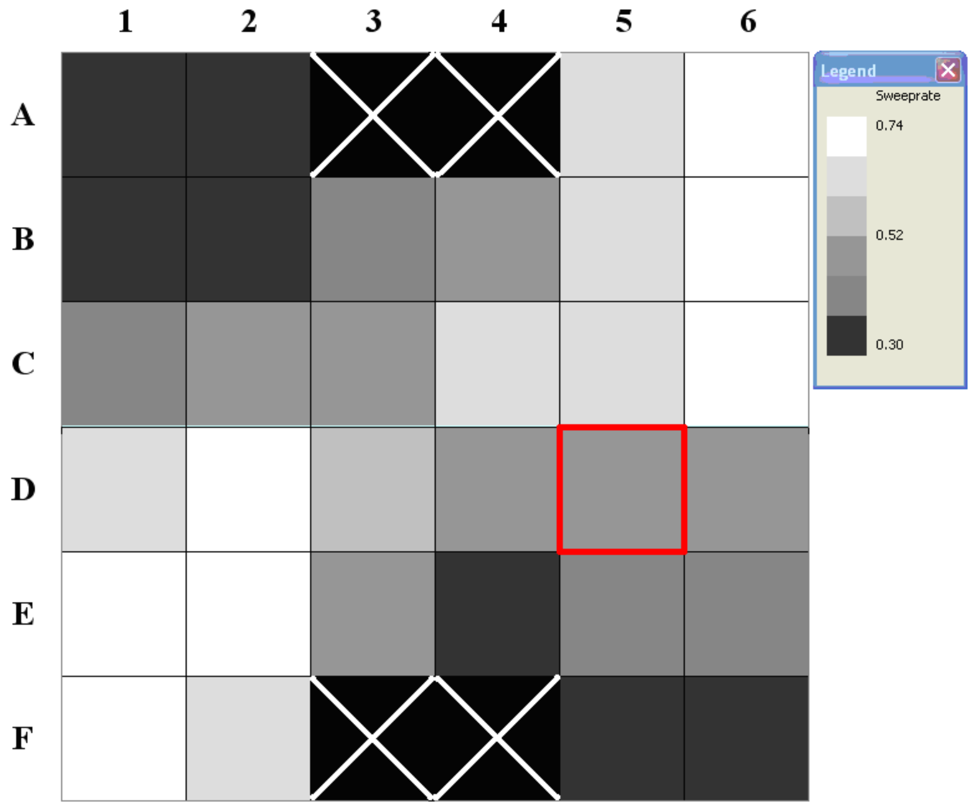


**Figure 19. Choke-point scenario, coverage rates**

The defender locates his platforms optimally as shown in Figure 20, with mission details listed in Table 11. Because the platforms search such a small area, this plan provides an objective value of 2.06, or a $P^D$ of 0.54. The platforms form a classic barrier patrol (Koopman 1980, pg. 196), but choose to search cells in the fourth column of the 4W grid, as those cells are easier to search.

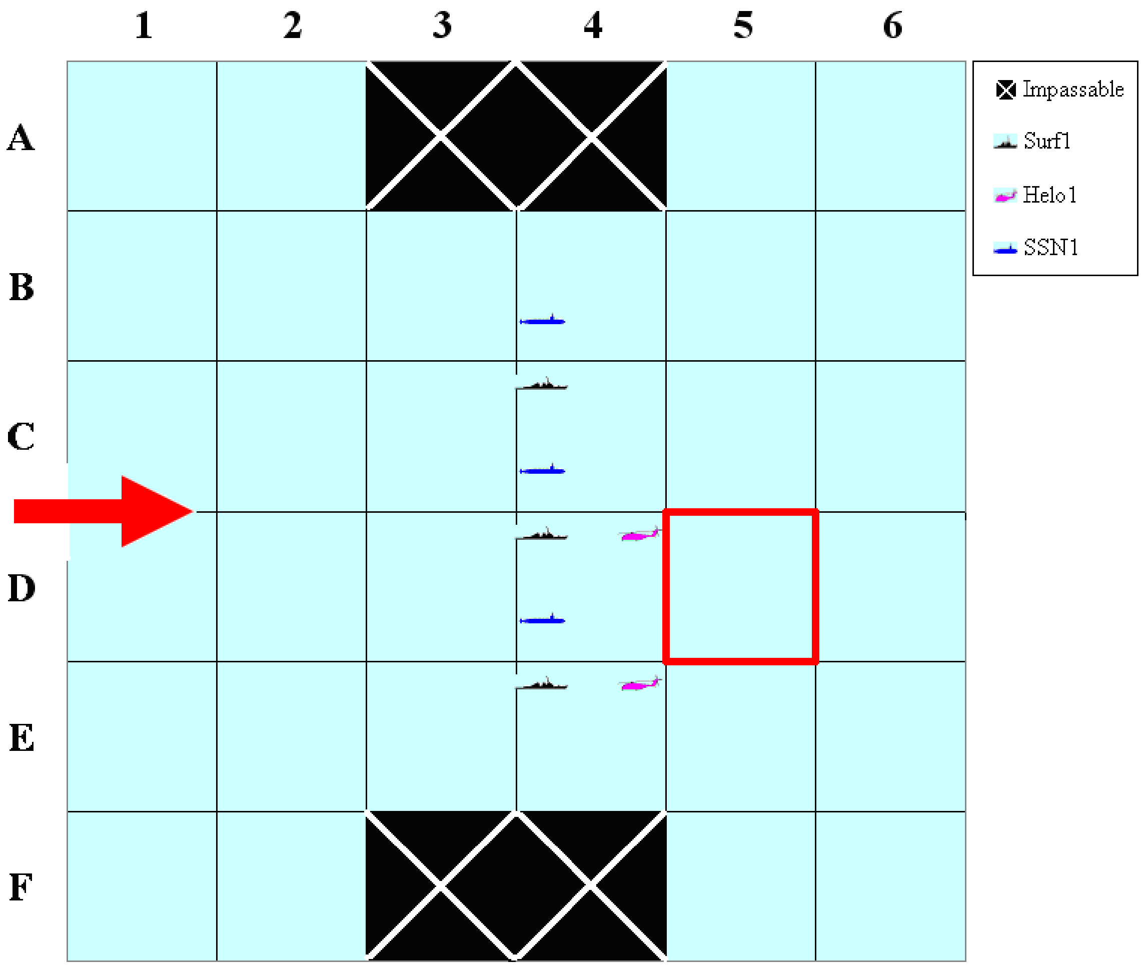


**Figure 20. Choke-point scenario, defensive platform positions**
The defender's platforms form a classic barrier patrol in easily searched water.

| Surf1 | active | Helo1 | active | SSN1 | active |
|---|---|---|---|---|---|
| **Cell** | **Time** | **Cell** | **Time** | **Cell** | **Time** |
| C4 | 1.61 | D4 | 0.30 | B4 | 2.35 |
| D4 | 1.89 | E4 | 3.70 | C4 | 0.70 |
| E4 | 0.50 | | | D4 | 0.95 |

**Table 11. Choke-point scenario, defensive platform mission assignments**
Sensor performance outweighs secrecy in this scenario, so SSN1 uses active sonar.

# IV. CONCLUSIONS AND RECOMMENDATIONS

This thesis has introduced a tri-level, defender-attacker/defender (D-A/D) model to prescribe optimal search plans for anti-submarine warfare (ASW) platforms seeking to protect a high-value unit (HVU, e.g., an aircraft carrier) from an intelligent enemy. In the model's first stage, the defender assigns mission tasking to his visible platforms (i.e., search platforms that are easily observed by the enemy using visual, acoustic, or by electronic-surveillance means). Given this information, then, the attacker and defender engage in a two-person, zero-sum, simultaneous-play game. This game assumes the attacker interprets the ocean environment as we do and reacts to visible platforms, and that he correctly infers which secret platforms we employ. (Secret platforms, which include submarines using passive sonar and aircraft deploying passive sonobuoys, benefit from their ability to hide from the enemy, but have limited detection ranges compared to visible platforms using active sonar.) As would be expected in this game, the defender randomizes the tasking of his secret platforms, while the attacker randomizes his approach routes to the HVU. This model, "G-TAMP," quantifies the value of secrecy by directing "flexible" platforms to remain secret or become visible, as required by the situation. In this way, the D-A/D model allows us to "shape" the battle space to our advantage with visible platforms in the first stage, and then exploit the secrecy of hidden platforms in the second stage.

Our model does not aim to replace current planning tools; rather, it leverages their capabilities to plan a coordinated ASW search effort among multiple platforms at the operational level. We propose integrating the capabilities of G-TAMP with the U.S. Navy's Undersea Warfare Decision Support System, developed by the Naval Undersea Warfare Center, which includes the STDA analysis tool, which analyzes the ocean environment and provides the requisite coverage-rate data for G-TAMP, and the planning tool ORP, which could transform our operational scheme into detailed track plans for platforms at the tactical level.

G-TAMP solves realistically sized problems in minutes using commercial off-the-shelf optimization (mixed-integer programming) software. We have suggested a decomposition technique that can reduce this time considerably; further study is required. If nothing else, the decomposition constructively suggests how to develop a heuristic that solves these problems without the need for any commercial optimization software. This would shorten solution times, and allow deployment of the model on any laptop computer.

We recommend further research into other applications for the D-A/D model. Because the ability to model an intelligent enemy and secret defenders plays an important role in ASW mission planning, we propose incorporating the D-A/D (or a similar model) into tactical-level decision aids such as ORP and ASW Screen Planner TDA.

The D-A/D model may also provide insight into areas other than ASW mission planning, such as homeland defense, in which a defender hopes to shape the battle space to his advantage, and then mitigate the consequences of an attacker's actions.

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# APPENDIX A: RELATIONSHIP BETWEEN PROBABILITY OF DETECTION AND PRESSURE

G-TAMP provides defensive plans that maximize the accumulated pressure along all SSK paths. Using notation from Chapter II, sections B and C, this appendix shows how this corresponds to minimizing the probability of non-detection of all SSKs.

Suppose that an SSK *i* remains stationary in any cell that it visits for some amount of time $t_i$. To calculate the probability of detection for this SSK in cell $g'$, $P^D_{g'i}$, we make the following assumptions: (1) each of the defender's platforms searches randomly within its assigned cells (this is a conservative assumption); (2) the pressure in cell $g'$ is exerted uniformly throughout the mission time horizon *T* (for example, after half of the mission time has elapsed, one-half of the final search pressure has been exerted in each cell). Then, by the random search formula (Washburn 2002, pg. 2-4),

$$P^D_{g'i} = 1 - e^{-f_{g'}(\mathbf{W},\mathbf{X})\frac{t_i}{T}}.$$

If we assume that $P^D_{g'i}$ is independent between cells, then the probability of detection for SSK *i* along an entire path, $P^D(i)$, is

$$P^D(i) = 1 - \prod_{(g,g')\in A_i} \left(1 - P^D_{g'i}\right) = 1 - \prod_{(g,g')\in A_i} e^{-f_{g'}(\mathbf{W},\mathbf{X})\frac{t_i}{T}}$$

where $A_i$ consists of all arcs traversed by the SSK.

The defender desires to minimize the probability of non-detection of all SSKs, $P^{ND}$, which, if we detect each submarine independently of the others, we can express as

$$P^{ND} = \prod_{i=1}^{n_{SUBS}} \left(1 - P^D(i)\right) = \prod_{i=1}^{n_{SUBS}} \prod_{(g,g')\in A_i} e^{-f_{g'}(\mathbf{W},\mathbf{X})\frac{t_i}{T}}.$$

Minimizing $P^{ND}$ is equivalent to minimizing the logarithm of $P^{ND}$,

$$\log P^{ND} = \sum_{i=1}^{n_{SUBS}} \sum_{(g,g')\in A_i} -f_{g'}(\mathbf{W},\mathbf{X}) \frac{t_i}{T}.$$

Because we sum over the paths of all SSKs, minimizing the above equation is equivalent to maximizing

$$\frac{t}{T} \sum_{(g,g')\in A} f_{g'}(\mathbf{W},\mathbf{X}) Y_{gg'},$$

where we assume each SSK spends the same amount of time $t$ in each cell. This is equivalent to maximizing the objective function in **SSCREEN**, (*a0′*). Therefore, minimizing the probability of non-detection of all SSKs corresponds to maximizing pressure along all SSK paths under simplifying assumptions. We use this measure of effectiveness so that prescribed solutions protect an HVU from attack by a single, or many, hostile SSKs.

Now, Washburn (2002, pp. 6-1 to 6-3) has shown how to improve this approximation. Because each approaching SSK does not remain stationary, we include a "dynamic enhancement" factor, which accounts for the increased probability of detection presented by a moving target. We use the previous assumptions, and the following: (3) the speed of each SSK is known (this is reasonable if an SSK is operating on battery power, as it surely would if attempting to approach an HVU stealthily); (4) the amount of time required for the attacker to transit a searched cell, $t_i$, is constant (this is approximately true for the geometry of the 4W grid).

Washburn presents a method for calculating a "dynamic enhancement" factor, the details of which will not be repeated here. For a target and searcher moving at speeds *U* and *V*, respectively, Table 12 shows several values of the dynamic enhancement factor, $\tilde{V}/V$.

| $U/V$ | $\tilde{V}/V$ |
|---|---|
| 0.0 | 1.00 |
| 0.2 | 1.01 |
| 0.5 | 1.06 |
| 1.0 | 1.27 |

**Table 12. Sample dynamic enhancement factor values**

This table shows the value of the dynamic enhancement factor for several target-searcher speed ratios. A value of 1.27 means that a platform searching at a speed $V$ for a target moving at the same speed achieves the same search performance as that platform searching at a speed of $1.27V$ for a stationary target (after Washburn 2002, pg. 6-3).

The dynamic enhancement factor expresses the improvement in search performance when a platform searches for a moving target compared to a stationary target. For example, for a target and searcher both moving at $V$ knots, $U/V=1$, so $\tilde{V}/V=1.27$. Therefore, $P^D$ would be the same for a searcher moving at a speed of $1.27V$ knots searching for a stationary target as for a searcher and target both moving at a speed of $V$ knots.

Once we have assumed a speed for the target SSK, we modify the equation for coverage rate to account for dynamic enhancement:

$$r_{psg} = \frac{(sweep\,width_{psg})(speed_p)(\tilde{V}/V)_p}{cell\,area_g},$$

where $(\tilde{V}/V)_p$ is the dynamic enhancement factor for platform $p$. By using this value for $r_{psg}$, we obtain better approximations for probability of detection.

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# APPENDIX B: SCENARIO DATA

The reader can reproduce any of our experiments from the data here. All distances are in nautical miles, all times are in hours, and all coverage rates are in $\text{hours}^{-1}$.

We use the following parameters in all examples:

| | |
|---|---|
| Cell dimension | 5 nm × 5 nm |
| $d$ | 0.01 |
| *battery* | 0.01 |
| $n_{SUBS}$ | 1 |
| $\overline{n}_{PLATS}$ | 10 |
| $u$ | 1 |
| $time_{pm}$ | 4 for all $p, m$ |
| SSK speed | 4 kts |
| Cell transit time | 1.5 hrs |

The remaining sets and data follow.

# A. EXAMPLE ONE: BASIC SCENARIO

## Sets

| $P_{VIS}$ | $P_{SECRET}$ | $P_{FLEX}$ | $P_{BASE}$ | $P_{TETH}$ | $PP_{TETH}$ | $G$ | $M$ | $M_{ps}$ |
|---|---|---|---|---|---|---|---|---|
| Surf1 | (empty) | SSN1 | Surf1 | Helo1 | Surf1.Helo1 | $g^+$ | {m1,…,m214} | {Surf1.m1.active,…,Surf1.m214.active} |
| Surf2 | | | | | | gA1 | | {Surf2.m1.active,…,Surf2.m214.active} |
| Surf3 | | | | | | gA2 (impassable) | | {Surf3.m1.active,…,Surf3.m214.active} |
| Helo1 | | | | | | gA3 | | {Helo1.m1.active,…,Helo1.m214.active} |
| P31 | | | | | | gA4 | | {P31.m1.active,…,P31.m214.active} |
| | | | | | | gA5 | | {SSN1.m1.active,…,SSN1.m214.active} |
| | | | | | | gA6 | | {SSN1.m1.passive,…,SSN1.m214.passive} |
| ***A*** | | | | | | gB1 | | |
| $g^+$.gA1 | gB2.gB3 | gC2.gD3 | gD3.gC4 | gE3.gF3 | | gB2 | | |
| $g^+$.gA3 | gB2.gC1 | gC2.$g^-$ | gD3.gD2 | gE3.gF4 | | gB3 | | |
| $g^+$.gA4 | gB2.gC2 | gC3.gB2 | gD3.gD4 | gE3.$g^-$ | | gB4 | | |
| $g^+$.gA5 | gB2.gC3 | gC3.gB3 | gD3.gE2 | gE4.gD3 | | gB5 | | |
| $g^+$.gA6 | gB2.$g^-$ | gC3.gB4 | gD3.gE3 | gE4.gD4 | | gB6 | | |
| $g^+$.gB1 | gB3.gA3 | gC3.gC2 | gD3.gE4 | gE4.gD5 | | gC1 | | |
| $g^+$.gB6 | gB3.gA4 | gC3.gC4 | gD4.gC3 | gE4.gE3 | | gC2 | | |
| $g^+$.gC1 | gB3.gB2 | gC3.gD2 | gD4.gC4 | gE4.gE5 | | gC3 (protected) | | |
| $g^+$.gC6 | gB3.gB4 | gC3.gD3 | gD4.gC5 | gE4.gF3 | | gC4 (protected) | | |
| $g^+$.gD1 | gB3.gC2 | gC3.gD4 | gD4.gD3 | gE4.gF4 | | gC5 | | |
| $g^+$.gD6 | gB3.gC3 | gC4.gB3 | gD4.gD5 | gE4.$g^-$ | | gC6 | | |
| $g^+$.gE1 | gB3.gC4 | gC4.gB4 | gD4.gE3 | gE5.gD4 | | gD1 | | |
| $g^+$.gE6 | gB3.$g^-$ | gC4.gB5 | gD4.gE4 | gE5.gD5 | | gD2 | | |
| $g^+$.gF1 | gB4.gA3 | gC4.gC3 | gD4.gE5 | gE5.gD6 | | gD3 (protected) | | |
| $g^+$.gF2 | gB4.gA4 | gC4.gC5 | gD5.gC4 | gE5.gE4 | | gD4 (protected) | | |
| $g^+$.gF3 | gB4.gA5 | gC4.gD3 | gD5.gC5 | gE5.gE6 | | gD5 | | |
| $g^+$.gF4 | gB4.gB3 | gC4.gD4 | gD5.gC6 | gE5.gF4 | | gD6 | | |
| gA1.gB1 | gB4.gB5 | gC4.gD5 | gD5.gD4 | gE5.$g^-$ | | gE1 | | |
| gA1.gB2 | gB4.gC3 | gC5.gB4 | gD5.gD6 | gE6.gD5 | | gE2 | | |
| gA2.gA1 | gB4.gC4 | gC5.gB5 | gD5.gE4 | gE6.gD6 | | gE3 | | |
| gA2.gA3 | gB4.gC5 | gC5.gB6 | gD5.gE5 | gE6.gE5 | | gE4 | | |
| gA2.gB1 | gB4.$g^-$ | gC5.gC4 | gD5.gE6 | gF1.gE1 | | gE5 | | |
| gA2.gB2 | gB5.gA4 | gC5.gC6 | gD5.$g^-$ | gF1.gE2 | | gE6 | | |
| gA2.gB3 | gB5.gA5 | gC5.gD4 | gD6.gC5 | gF1.gF2 | | gF1 | | |
| gA3.gA4 | gB5.gA6 | gC5.gD5 | gD6.gC6 | gF2.gE1 | | gF2 | | |
| gA3.gB2 | gB5.gB4 | gC5.gD6 | gD6.gD5 | gF2.gE2 | | gF3 | | |
| gA3.gB3 | gB5.gB6 | gC5.$g^-$ | gD6.gE5 | gF2.gE3 | | gF4 | | |
| gA3.gB4 | gB5.gC4 | gC6.gB5 | gD6.gE6 | gF2.gF1 | | gF5 (impassable) | | |
| gA4.gA3 | gB5.gC5 | gC6.gB6 | gE1.gD1 | gF2.gF3 | | gF6 (impassable) | | |
| gA4.gA5 | gB5.gC6 | gC6.gC5 | gE1.gD2 | gF3.gE2 | | $g^-$ | | |
| gA4.gB3 | gB5.$g^-$ | gC6.gD5 | gE1.gE2 | gF3.gE3 | | | | |
| gA4.gB4 | gB6.gA5 | gC6.gD6 | gE1.gF1 | gF3.gE4 | | | | |
| gA4.gB5 | gB6.gA6 | gD1.gC1 | gE1.gF2 | gF3.gF2 | | | | |
| gA5.gA4 | gB6.gB5 | gD1.gC2 | gE2.gD1 | gF3.gF4 | | | | |
| gA5.gA6 | gB6.gC5 | gD1.gD2 | gE2.gD2 | gF4.gE3 | | | | |
| gA5.gB4 | gB6.gC6 | gD1.gE1 | gE2.gD3 | gF4.gE4 | | | | |
| gA5.gB5 | gC1.gB1 | gD1.gE2 | gE2.gE1 | gF4.gE5 | | | | |
| gA5.gB6 | gC1.gB2 | gD2.gC1 | gE2.gE3 | gF4.gF3 | | | | |
| gA6.gA5 | gC1.gC2 | gD2.gC2 | gE2.gF1 | gF5.gE4 | | | | |
| gA6.gB5 | gC1.gD1 | gD2.gC3 | gE2.gF2 | gF5.gE5 | | | | |
| gA6.gB6 | gC1.gD2 | gD2.gD1 | gE2.gF3 | gF5.gE6 | | | | |
| gB1.gA1 | gC2.gB1 | gD2.gD3 | gE2.$g^-$ | gF5.gF4 | | | | |
| gB1.gB2 | gC2.gB2 | gD2.gE1 | gE3.gD2 | gF6.gE5 | | | | |
| gB1.gC1 | gC2.gB3 | gD2.gE2 | gE3.gD3 | gF6.gE6 | | | | |
| gB1.gC2 | gC2.gC1 | gD2.gE3 | gE3.gD4 | | | | | |
| gB2.gA1 | gC2.gC3 | gD2.$g^-$ | gE3.gE2 | | | | | |
| gB2.gA3 | gC2.gD1 | gD3.gC2 | gE3.gE4 | | | | | |
| gB2.gB1 | gC2.gD2 | gD3.gC3 | gE3.gF2 | | | | | |

***$G_{pm}$ (for all p in P)***

| | | | | | |
|---|---|---|---|---|---|
| p.m1.gA1 | p.m39.gC1 | p.m61.gA4 | p.m83.gE6 | p.m99.gD2 | p.m114.gC1 |
| p.m2.gA3 | p.m40.gB2 | p.m62.gA4 | p.m84.gF1 | p.m99.gE2 | p.m114.gC2 |
| p.m3.gA4 | p.m40.gC2 | p.m62.gA5 | p.m84.gF2 | p.m100.gC3 | p.m114.gC3 |
| p.m4.gA5 | p.m41.gB3 | p.m63.gA5 | p.m85.gF2 | p.m100.gD3 | p.m115.gC2 |
| p.m5.gA6 | p.m41.gC3 | p.m63.gA6 | p.m85.gF3 | p.m100.gE3 | p.m115.gC3 |
| p.m6.gB1 | p.m42.gB4 | p.m64.gB1 | p.m86.gF3 | p.m101.gC4 | p.m115.gC4 |
| p.m7.gB2 | p.m42.gC4 | p.m64.gB2 | p.m86.gF4 | p.m101.gD4 | p.m116.gC3 |
| p.m8.gB3 | p.m43.gB5 | p.m65.gB2 | p.m87.gA1 | p.m101.gE4 | p.m116.gC4 |
| p.m9.gB4 | p.m43.gC5 | p.m65.gB3 | p.m87.gB1 | p.m102.gC5 | p.m116.gC5 |
| p.m10.gB5 | p.m44.gB6 | p.m66.gB3 | p.m87.gC1 | p.m102.gD5 | p.m117.gC4 |
| p.m11.gB6 | p.m44.gC6 | p.m66.gB4 | p.m88.gA3 | p.m102.gE5 | p.m117.gC5 |
| p.m12.gC1 | p.m45.gC1 | p.m67.gB4 | p.m88.gB3 | p.m103.gC6 | p.m117.gC6 |
| p.m13.gC2 | p.m45.gD1 | p.m67.gB5 | p.m88.gC3 | p.m103.gD6 | p.m118.gD1 |
| p.m14.gC3 | p.m46.gC2 | p.m68.gB5 | p.m89.gA4 | p.m103.gE6 | p.m118.gD2 |
| p.m15.gC4 | p.m46.gD2 | p.m68.gB6 | p.m89.gB4 | p.m104.gD1 | p.m118.gD3 |
| p.m16.gC5 | p.m47.gC3 | p.m69.gC1 | p.m89.gC4 | p.m104.gE1 | p.m119.gD2 |
| p.m17.gC6 | p.m47.gD3 | p.m69.gC2 | p.m90.gA5 | p.m104.gF1 | p.m119.gD3 |
| p.m18.gD1 | p.m48.gC4 | p.m70.gC2 | p.m90.gB5 | p.m105.gD2 | p.m119.gD4 |
| p.m19.gD2 | p.m48.gD4 | p.m70.gC3 | p.m90.gC5 | p.m105.gE2 | p.m120.gD3 |
| p.m20.gD3 | p.m49.gC5 | p.m71.gC3 | p.m91.gA6 | p.m105.gF2 | p.m120.gD4 |
| p.m21.gD4 | p.m49.gD5 | p.m71.gC4 | p.m91.gB6 | p.m106.gD3 | p.m120.gD5 |
| p.m22.gD5 | p.m50.gC6 | p.m72.gC4 | p.m91.gC6 | p.m106.gE3 | p.m121.gD4 |
| p.m23.gD6 | p.m50.gD6 | p.m72.gC5 | p.m92.gB1 | p.m106.gF3 | p.m121.gD5 |
| p.m24.gE1 | p.m51.gD1 | p.m73.gC5 | p.m92.gC1 | p.m107.gD4 | p.m121.gD6 |
| p.m25.gE2 | p.m51.gE1 | p.m73.gC6 | p.m92.gD1 | p.m107.gE4 | p.m122.gE1 |
| p.m26.gE3 | p.m52.gD2 | p.m74.gD1 | p.m93.gB2 | p.m107.gF4 | p.m122.gE2 |
| p.m27.gE4 | p.m52.gE2 | p.m74.gD2 | p.m93.gC2 | p.m108.gA3 | p.m122.gE3 |
| p.m28.gE5 | p.m53.gD3 | p.m75.gD2 | p.m93.gD2 | p.m108.gA4 | p.m123.gE2 |
| p.m29.gE6 | p.m53.gE3 | p.m75.gD3 | p.m94.gB3 | p.m108.gA5 | p.m123.gE3 |
| p.m30.gF1 | p.m54.gD4 | p.m76.gD3 | p.m94.gC3 | p.m109.gA4 | p.m123.gE4 |
| p.m31.gF2 | p.m54.gE4 | p.m76.gD4 | p.m94.gD3 | p.m109.gA5 | p.m124.gE3 |
| p.m32.gF3 | p.m55.gD5 | p.m77.gD4 | p.m95.gB4 | p.m109.gA6 | p.m124.gE4 |
| p.m33.gF4 | p.m55.gE5 | p.m77.gD5 | p.m95.gC4 | p.m110.gB1 | p.m124.gE5 |
| p.m34.gA1 | p.m56.gD6 | p.m78.gD5 | p.m95.gD4 | p.m110.gB2 | p.m125.gE4 |
| p.m34.gB1 | p.m56.gE6 | p.m78.gD6 | p.m96.gB5 | p.m110.gB3 | p.m125.gE5 |
| p.m35.gA3 | p.m57.gE1 | p.m79.gE1 | p.m96.gC5 | p.m111.gB2 | p.m125.gE6 |
| p.m35.gB3 | p.m57.gF1 | p.m79.gE2 | p.m96.gD5 | p.m111.gB3 | p.m126.gF1 |
| p.m36.gA4 | p.m58.gE2 | p.m80.gE2 | p.m97.gB6 | p.m111.gB4 | p.m126.gF2 |
| p.m36.gB4 | p.m58.gF2 | p.m80.gE3 | p.m97.gC6 | p.m112.gB3 | p.m126.gF3 |
| p.m37.gA5 | p.m59.gE3 | p.m81.gE3 | p.m97.gD6 | p.m112.gB4 | p.m127.gF2 |
| p.m37.gB5 | p.m59.gF3 | p.m81.gE4 | p.m98.gC1 | p.m112.gB5 | p.m127.gF3 |
| p.m38.gA6 | p.m60.gE4 | p.m82.gE4 | p.m98.gD1 | p.m113.gB4 | p.m127.gF4 |
| p.m38.gB6 | p.m60.gF4 | p.m82.gE5 | p.m98.gE1 | p.m113.gB5 | p.m128.gA3 |
| p.m39.gB1 | p.m61.gA3 | p.m83.gE5 | p.m99.gC2 | p.m113.gB6 | p.m128.gA4 |

***$G_{pm}$ (for all p in P) (continued)***

| | | | | | |
|---|---|---|---|---|---|
| p.m128.gB3 | p.m143.gD4 | p.m158.gB5 | p.m172.gB3 | p.m187.gD3 | p.m202.gC1 |
| p.m129.gA4 | p.m143.gE3 | p.m158.gC5 | p.m173.gB3 | p.m187.gE3 | p.m202.gC2 |
| p.m129.gA5 | p.m144.gD4 | p.m158.gC6 | p.m173.gA4 | p.m188.gE3 | p.m202.gD2 |
| p.m129.gB4 | p.m144.gD5 | p.m159.gC1 | p.m173.gB4 | p.m188.gD4 | p.m203.gC2 |
| p.m130.gA5 | p.m144.gE4 | p.m159.gD1 | p.m174.gB4 | p.m188.gE4 | p.m203.gC3 |
| p.m130.gA6 | p.m145.gD5 | p.m159.gD2 | p.m174.gA5 | p.m189.gE4 | p.m203.gD3 |
| p.m130.gB5 | p.m145.gD6 | p.m160.gC2 | p.m174.gB5 | p.m189.gD5 | p.m204.gC3 |
| p.m131.gB1 | p.m145.gE5 | p.m160.gD2 | p.m175.gB5 | p.m189.gE5 | p.m204.gC4 |
| p.m131.gB2 | p.m146.gE1 | p.m160.gD3 | p.m175.gA6 | p.m190.gE5 | p.m204.gD4 |
| p.m131.gC1 | p.m146.gE2 | p.m161.gC3 | p.m175.gB6 | p.m190.gD6 | p.m205.gC4 |
| p.m132.gB2 | p.m146.gF1 | p.m161.gD3 | p.m176.gC1 | p.m190.gE6 | p.m205.gC5 |
| p.m132.gB3 | p.m147.gE2 | p.m161.gD4 | p.m176.gB2 | p.m191.gF1 | p.m205.gD5 |
| p.m132.gC2 | p.m147.gE3 | p.m162.gC4 | p.m176.gC2 | p.m191.gE2 | p.m206.gC5 |
| p.m133.gB3 | p.m147.gF2 | p.m162.gD4 | p.m177.gC2 | p.m191.gF2 | p.m206.gC6 |
| p.m133.gB4 | p.m148.gE3 | p.m162.gD5 | p.m177.gB3 | p.m192.gF2 | p.m206.gD6 |
| p.m133.gC3 | p.m148.gE4 | p.m163.gC5 | p.m177.gC3 | p.m192.gE3 | p.m207.gD1 |
| p.m134.gB4 | p.m148.gF3 | p.m163.gD5 | p.m178.gC3 | p.m192.gF3 | p.m207.gD2 |
| p.m134.gB5 | p.m149.gE4 | p.m163.gD6 | p.m178.gB4 | p.m193.gF3 | p.m207.gE2 |
| p.m134.gC4 | p.m149.gE5 | p.m164.gD1 | p.m178.gC4 | p.m193.gE4 | p.m208.gD2 |
| p.m135.gB5 | p.m149.gF4 | p.m164.gE1 | p.m179.gC4 | p.m193.gF4 | p.m208.gD3 |
| p.m135.gB6 | p.m150.gA1 | p.m164.gE2 | p.m179.gB5 | p.m194.gA3 | p.m208.gE3 |
| p.m135.gC5 | p.m150.gB1 | p.m165.gD2 | p.m179.gC5 | p.m194.gA4 | p.m209.gD3 |
| p.m136.gC1 | p.m150.gB2 | p.m165.gE2 | p.m180.gC5 | p.m194.gB4 | p.m209.gD4 |
| p.m136.gC2 | p.m151.gA3 | p.m165.gE3 | p.m180.gB6 | p.m195.gA4 | p.m209.gE4 |
| p.m136.gD1 | p.m151.gB3 | p.m166.gD3 | p.m180.gC6 | p.m195.gA5 | p.m210.gD4 |
| p.m137.gC2 | p.m151.gB4 | p.m166.gE3 | p.m181.gD1 | p.m195.gB5 | p.m210.gD5 |
| p.m137.gC3 | p.m152.gA4 | p.m166.gE4 | p.m181.gC2 | p.m196.gA5 | p.m210.gE5 |
| p.m137.gD2 | p.m152.gB4 | p.m167.gD4 | p.m181.gD2 | p.m196.gA6 | p.m211.gD5 |
| p.m138.gC3 | p.m152.gB5 | p.m167.gE4 | p.m182.gD2 | p.m196.gB6 | p.m211.gD6 |
| p.m138.gC4 | p.m153.gA5 | p.m167.gE5 | p.m182.gC3 | p.m197.gB1 | p.m211.gE6 |
| p.m138.gD3 | p.m153.gB5 | p.m168.gD5 | p.m182.gD3 | p.m197.gB2 | p.m212.gE1 |
| p.m139.gC4 | p.m153.gB6 | p.m168.gE5 | p.m183.gD3 | p.m197.gC2 | p.m212.gE2 |
| p.m139.gC5 | p.m154.gB1 | p.m168.gE6 | p.m183.gC4 | p.m198.gB2 | p.m212.gF2 |
| p.m139.gD4 | p.m154.gC1 | p.m169.gE1 | p.m183.gD4 | p.m198.gB3 | p.m213.gE2 |
| p.m140.gC5 | p.m154.gC2 | p.m169.gF1 | p.m184.gD4 | p.m198.gC3 | p.m213.gE3 |
| p.m140.gC6 | p.m155.gB2 | p.m169.gF2 | p.m184.gC5 | p.m199.gB3 | p.m213.gF3 |
| p.m140.gD5 | p.m155.gC2 | p.m170.gE2 | p.m184.gD5 | p.m199.gB4 | p.m214.gE3 |
| p.m141.gD1 | p.m155.gC3 | p.m170.gF2 | p.m185.gD5 | p.m199.gC4 | p.m214.gE4 |
| p.m141.gD2 | p.m156.gB3 | p.m170.gF3 | p.m185.gC6 | p.m200.gB4 | p.m214.gF4 |
| p.m141.gE1 | p.m156.gC3 | p.m171.gE3 | p.m185.gD6 | p.m200.gB5 | |
| p.m142.gD2 | p.m156.gC4 | p.m171.gF3 | p.m186.gE1 | p.m200.gC5 | |
| p.m142.gD3 | p.m157.gB4 | p.m171.gF4 | p.m186.gD2 | p.m201.gB5 | |
| p.m142.gE2 | p.m157.gC4 | p.m172.gB2 | p.m186.gE2 | p.m201.gB6 | |
| p.m143.gD3 | p.m157.gC5 | p.m172.gA3 | p.m187.gE2 | p.m201.gC6 | |

## Data

***dist*** $_q$

| | |
|---|---|
| gA1 | 10.0 |
| gA2 | 10.0 |
| gA3 | 10.0 |
| gA4 | 10.0 |
| gA5 | 10.0 |
| gA6 | 10.0 |
| gB1 | 10.0 |
| gB2 | 5.0 |
| gB3 | 5.0 |
| gB4 | 5.0 |
| gB5 | 5.0 |
| gB6 | 10.0 |
| gC1 | 10.0 |
| gC2 | 5.0 |
| gC3 | 0.0 |
| gC4 | 0.0 |
| gC5 | 5.0 |
| gC6 | 10.0 |
| gD1 | 10.0 |
| gD2 | 5.0 |
| gD3 | 0.0 |
| gD4 | 0.0 |
| gD5 | 5.0 |
| gD6 | 10.0 |
| gE1 | 10.0 |
| gE2 | 5.0 |
| gE3 | 5.0 |
| gE4 | 5.0 |
| gE5 | 5.0 |
| gE6 | 10.0 |
| gF1 | 10.0 |
| gF2 | 10.0 |
| gF3 | 10.0 |
| gF4 | 10.0 |
| gF5 | 10.0 |
| gF6 | 10.0 |

***teth_range*** $_{p''}$

| | |
|---|---|
| Helo1 | 10.0 |

***trans*** $_p$

| | |
|---|---|
| Surf1 | 0.5 |
| Surf2 | 0.5 |
| Surf3 | 0.5 |
| Helo1 | 0.1 |
| P31 | 0.1 |
| SSN1 | 0.7 |

***r*** $_{psq}$

| | Surf1.active | Surf2.active | Surf3.active | Helo1.active | P31.active | SSN1.passive | SSN1.active |
|---|---|---|---|---|---|---|---|
| gA1 | 0.42 | 0.42 | 0.42 | 0.5 | 0.55 | 0.45 | 0.63 |
| gA2 | 0.42 | 0.42 | 0.42 | 0.5 | 0.55 | 0.45 | 0.63 |
| gA3 | 0.56 | 0.56 | 0.56 | 0.67 | 0.73 | 0.6 | 0.84 |
| gA4 | 0.7 | 0.7 | 0.7 | 0.84 | 0.91 | 0.75 | 1.05 |
| gA5 | 0.91 | 0.91 | 0.91 | 1.09 | 1.18 | 0.97 | 1.36 |
| gA6 | 0.95 | 0.95 | 0.95 | 1.14 | 1.24 | 1.02 | 1.43 |
| gB1 | 0.42 | 0.42 | 0.42 | 0.5 | 0.55 | 0.45 | 0.63 |
| gB2 | 0.49 | 0.49 | 0.49 | 0.59 | 0.64 | 0.53 | 0.74 |
| gB3 | 0.56 | 0.56 | 0.56 | 0.67 | 0.73 | 0.6 | 0.84 |
| gB4 | 0.7 | 0.7 | 0.7 | 0.84 | 0.91 | 0.75 | 1.05 |
| gB5 | 0.88 | 0.88 | 0.88 | 1.05 | 1.14 | 0.94 | 1.32 |
| gB6 | 1.03 | 1.03 | 1.03 | 1.24 | 1.34 | 1.11 | 1.55 |
| gC1 | 0.56 | 0.56 | 0.56 | 0.67 | 0.73 | 0.6 | 0.84 |
| gC2 | 0.63 | 0.63 | 0.63 | 0.76 | 0.82 | 0.68 | 0.95 |
| gC3 | 0.7 | 0.7 | 0.7 | 0.84 | 0.91 | 0.75 | 1.05 |
| gC4 | 0.84 | 0.84 | 0.84 | 1.01 | 1.09 | 0.9 | 1.26 |
| gC5 | 0.91 | 0.91 | 0.91 | 1.1 | 1.19 | 0.98 | 1.37 |
| gC6 | 0.97 | 0.97 | 0.96 | 1.16 | 1.26 | 1.03 | 1.45 |
| gD1 | 0.91 | 0.91 | 0.91 | 1.09 | 1.18 | 0.98 | 1.37 |
| gD2 | 1.01 | 1.01 | 1.01 | 1.22 | 1.32 | 1.08 | 1.52 |
| gD3 | 0.77 | 0.77 | 0.77 | 0.92 | 1 | 0.83 | 1.16 |
| gD4 | 0.63 | 0.63 | 0.63 | 0.76 | 0.82 | 0.68 | 0.95 |
| gD5 | 0.7 | 0.7 | 0.7 | 0.84 | 0.91 | 0.75 | 1.05 |
| gD6 | 0.7 | 0.7 | 0.7 | 0.84 | 0.91 | 0.75 | 1.05 |
| gE1 | 0.99 | 0.99 | 0.99 | 1.19 | 1.29 | 1.06 | 1.48 |
| gE2 | 1.03 | 1.03 | 1.03 | 1.24 | 1.34 | 1.1 | 1.55 |
| gE3 | 0.63 | 0.63 | 0.63 | 0.76 | 0.82 | 0.68 | 0.95 |
| gE4 | 0.49 | 0.49 | 0.49 | 0.59 | 0.64 | 0.53 | 0.74 |
| gE5 | 0.56 | 0.56 | 0.56 | 0.67 | 0.73 | 0.6 | 0.84 |
| gE6 | 0.56 | 0.56 | 0.56 | 0.67 | 0.73 | 0.6 | 0.84 |
| gF1 | 0.95 | 0.95 | 0.95 | 1.14 | 1.24 | 1.02 | 1.43 |
| gF2 | 0.88 | 0.88 | 0.88 | 1.05 | 1.14 | 0.94 | 1.31 |
| gF3 | 0.56 | 0.56 | 0.56 | 0.67 | 0.73 | 0.6 | 0.84 |
| gF4 | 0.42 | 0.42 | 0.42 | 0.5 | 0.55 | 0.45 | 0.63 |
| gF5 | 0.42 | 0.42 | 0.42 | 0.5 | 0.55 | 0.45 | 0.63 |
| gF6 | 0.42 | 0.42 | 0.42 | 0.5 | 0.55 | 0.45 | 0.63 |

***range*** $_{m'm''}$

Determined from table below

e.g., for range(m1, m128), take the minimum of the distances (gA1, gA3), (gA1, gA4), (gA1, gB3), which are 10, 15, 11.2, respectively. Therefore, range(m1, m128) is 10.

| | gA1 | gA2 | gA3 | gA4 | gA5 | gA6 | gB1 | gB2 | gB3 | gB4 | gB5 | gB6 | gC1 | gC2 | gC3 | gC4 | gC5 | gC6 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| gA1 | 0 | 5 | 10 | 15 | 20 | 25 | 5 | 7.1 | 11.2 | 15.8 | 20.6 | 25.5 | 10 | 11.2 | 14.1 | 18 | 22.4 | 26.9 |
| gA2 | 5 | 0 | 5 | 10 | 15 | 20 | 7.1 | 5 | 7.1 | 11.2 | 15.8 | 20.6 | 11.2 | 10 | 11.2 | 14.1 | 18 | 22.4 |
| gA3 | 10 | 5 | 0 | 5 | 10 | 15 | 11.2 | 7.1 | 5 | 7.1 | 11.2 | 15.8 | 14.1 | 11.2 | 10 | 11.2 | 14.1 | 18 |
| gA4 | 15 | 10 | 5 | 0 | 5 | 10 | 15.8 | 11.2 | 7.1 | 5 | 7.1 | 11.2 | 18 | 14.1 | 11.2 | 10 | 11.2 | 14.1 |
| gA5 | 20 | 15 | 10 | 5 | 0 | 5 | 20.6 | 15.8 | 11.2 | 7.1 | 5 | 7.1 | 22.4 | 18 | 14.1 | 11.2 | 10 | 11.2 |
| gA6 | 25 | 20 | 15 | 10 | 5 | 0 | 25.5 | 20.6 | 15.8 | 11.2 | 7.1 | 5 | 26.9 | 22.4 | 18 | 14.1 | 11.2 | 10 |
| gB1 | 5 | 7.1 | 11.2 | 15.8 | 20.6 | 25.5 | 0 | 5 | 10 | 15 | 20 | 25 | 5 | 7.1 | 11.2 | 15.8 | 20.6 | 25.5 |
| gB2 | 7.1 | 5 | 7.1 | 11.2 | 15.8 | 20.6 | 5 | 0 | 5 | 10 | 15 | 20 | 7.1 | 5 | 7.1 | 11.2 | 15.8 | 20.6 |
| gB3 | 11.2 | 7.1 | 5 | 7.1 | 11.2 | 15.8 | 10 | 5 | 0 | 5 | 10 | 15 | 11.2 | 7.1 | 5 | 7.1 | 11.2 | 15.8 |
| gB4 | 15.8 | 11.2 | 7.1 | 5 | 7.1 | 11.2 | 15 | 10 | 5 | 0 | 5 | 10 | 15.8 | 11.2 | 7.1 | 5 | 7.1 | 11.2 |
| gB5 | 20.6 | 15.8 | 11.2 | 7.1 | 5 | 7.1 | 20 | 15 | 10 | 5 | 0 | 5 | 20.6 | 15.8 | 11.2 | 7.1 | 5 | 7.1 |
| gB6 | 25.5 | 20.6 | 15.8 | 11.2 | 7.1 | 5 | 25 | 20 | 15 | 10 | 5 | 0 | 25.5 | 20.6 | 15.8 | 11.2 | 7.1 | 5 |
| gC1 | 10 | 11.2 | 14.1 | 18 | 22.4 | 26.9 | 5 | 7.1 | 11.2 | 15.8 | 20.6 | 25.5 | 0 | 5 | 10 | 15 | 20 | 25 |
| gC2 | 11.2 | 10 | 11.2 | 14.1 | 18 | 22.4 | 7.1 | 5 | 7.1 | 11.2 | 15.8 | 20.6 | 5 | 0 | 5 | 10 | 15 | 20 |
| gC3 | 14.1 | 11.2 | 10 | 11.2 | 14.1 | 18 | 11.2 | 7.1 | 5 | 7.1 | 11.2 | 15.8 | 10 | 5 | 0 | 5 | 10 | 15 |
| gC4 | 18 | 14.1 | 11.2 | 10 | 11.2 | 14.1 | 15.8 | 11.2 | 7.1 | 5 | 7.1 | 11.2 | 15 | 10 | 5 | 0 | 5 | 10 |
| gC5 | 22.4 | 18 | 14.1 | 11.2 | 10 | 11.2 | 20.6 | 15.8 | 11.2 | 7.1 | 5 | 7.1 | 20 | 15 | 10 | 5 | 0 | 5 |
| gC6 | 26.9 | 22.4 | 18 | 14.1 | 11.2 | 10 | 25.5 | 20.6 | 15.8 | 11.2 | 7.1 | 5 | 25 | 20 | 15 | 10 | 5 | 0 |
| gD1 | 15 | 15.8 | 18 | 21.2 | 25 | 29.2 | 10 | 11.2 | 14.1 | 18 | 22.4 | 26.9 | 5 | 7.1 | 11.2 | 15.8 | 20.6 | 25.5 |
| gD2 | 15.8 | 15 | 15.8 | 18 | 21.2 | 25 | 11.2 | 10 | 11.2 | 14.1 | 18 | 22.4 | 7.1 | 5 | 7.1 | 11.2 | 15.8 | 20.6 |
| gD3 | 18 | 15.8 | 15 | 15.8 | 18 | 21.2 | 14.1 | 11.2 | 10 | 11.2 | 14.1 | 18 | 11.2 | 7.1 | 5 | 7.1 | 11.2 | 15.8 |
| gD4 | 21.2 | 18 | 15.8 | 15 | 15.8 | 18 | 18 | 14.1 | 11.2 | 10 | 11.2 | 14.1 | 15.8 | 11.2 | 7.1 | 5 | 7.1 | 11.2 |
| gD5 | 25 | 21.2 | 18 | 15.8 | 15 | 15.8 | 22.4 | 18 | 14.1 | 11.2 | 10 | 11.2 | 20.6 | 15.8 | 11.2 | 7.1 | 5 | 7.1 |
| gD6 | 29.2 | 25 | 21.2 | 18 | 15.8 | 15 | 26.9 | 22.4 | 18 | 14.1 | 11.2 | 10 | 25.5 | 20.6 | 15.8 | 11.2 | 7.1 | 5 |
| gE1 | 20 | 20.6 | 22.4 | 25 | 28.3 | 32 | 15 | 15.8 | 18 | 21.2 | 25 | 29.2 | 10 | 11.2 | 14.1 | 18 | 22.4 | 26.9 |
| gE2 | 20.6 | 20 | 20.6 | 22.4 | 25 | 28.3 | 15.8 | 15 | 15.8 | 18 | 21.2 | 25 | 11.2 | 10 | 11.2 | 14.1 | 18 | 22.4 |
| gE3 | 22.4 | 20.6 | 20 | 20.6 | 22.4 | 25 | 18 | 15.8 | 15 | 15.8 | 18 | 21.2 | 14.1 | 11.2 | 10 | 11.2 | 14.1 | 18 |
| gE4 | 25 | 22.4 | 20.6 | 20 | 20.6 | 22.4 | 21.2 | 18 | 15.8 | 15 | 15.8 | 18 | 18 | 14.1 | 11.2 | 10 | 11.2 | 14.1 |
| gE5 | 28.3 | 25 | 22.4 | 20.6 | 20 | 20.6 | 25 | 21.2 | 18 | 15.8 | 15 | 15.8 | 22.4 | 18 | 14.1 | 11.2 | 10 | 11.2 |
| gE6 | 32 | 28.3 | 25 | 22.4 | 20.6 | 20 | 29.2 | 25 | 21.2 | 18 | 15.8 | 15 | 26.9 | 22.4 | 18 | 14.1 | 11.2 | 10 |
| gF1 | 25 | 25.5 | 26.9 | 29.2 | 32 | 35.4 | 20 | 20.6 | 22.4 | 25 | 28.3 | 32 | 15 | 15.8 | 18 | 21.2 | 25 | 29.2 |
| gF2 | 25.5 | 25 | 25.5 | 26.9 | 29.2 | 32 | 20.6 | 20 | 20.6 | 22.4 | 25 | 28.3 | 15.8 | 15 | 15.8 | 18 | 21.2 | 25 |
| gF3 | 26.9 | 25.5 | 25 | 25.5 | 26.9 | 29.2 | 22.4 | 20.6 | 20 | 20.6 | 22.4 | 25 | 18 | 15.8 | 15 | 15.8 | 18 | 21.2 |
| gF4 | 29.2 | 26.9 | 25.5 | 25 | 25.5 | 26.9 | 25 | 22.4 | 20.6 | 20 | 20.6 | 22.4 | 21.2 | 18 | 15.8 | 15 | 15.8 | 18 |
| gF5 | 32 | 29.2 | 26.9 | 25.5 | 25 | 25.5 | 28.3 | 25 | 22.4 | 20.6 | 20 | 20.6 | 25 | 21.2 | 18 | 15.8 | 15 | 15.8 |
| gF6 | 35.4 | 32 | 29.2 | 26.9 | 25.5 | 25 | 32 | 28.3 | 25 | 22.4 | 20.6 | 20 | 29.2 | 25 | 21.2 | 18 | 15.8 | 15 |

***range $_{m'm''}$ (continued)***

| | gD1 | gD2 | gD3 | gD4 | gD5 | gD6 | gE1 | gE2 | gE3 | gE4 | gE5 | gE6 | gF1 | gF2 | gF3 | gF4 | gF5 | gF6 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| gA1 | 15 | 15.8 | 18 | 21.2 | 25 | 29.2 | 20 | 20.6 | 22.4 | 25 | 28.3 | 32 | 25 | 25.5 | 26.9 | 29.2 | 32 | 35.4 |
| gA2 | 15.8 | 15 | 15.8 | 18 | 21.2 | 25 | 20.6 | 20 | 20.6 | 22.4 | 25 | 28.3 | 25.5 | 25 | 25.5 | 26.9 | 29.2 | 32 |
| gA3 | 18 | 15.8 | 15 | 15.8 | 18 | 21.2 | 22.4 | 20.6 | 20 | 20.6 | 22.4 | 25 | 26.9 | 25.5 | 25 | 25.5 | 26.9 | 29.2 |
| gA4 | 21.2 | 18 | 15.8 | 15 | 15.8 | 18 | 25 | 22.4 | 20.6 | 20 | 20.6 | 22.4 | 29.2 | 26.9 | 25.5 | 25 | 25.5 | 26.9 |
| gA5 | 25 | 21.2 | 18 | 15.8 | 15 | 15.8 | 28.3 | 25 | 22.4 | 20.6 | 20 | 20.6 | 32 | 29.2 | 26.9 | 25.5 | 25 | 25.5 |
| gA6 | 29.2 | 25 | 21.2 | 18 | 15.8 | 15 | 32 | 28.3 | 25 | 22.4 | 20.6 | 20 | 35.4 | 32 | 29.2 | 26.9 | 25.5 | 25 |
| gB1 | 10 | 11.2 | 14.1 | 18 | 22.4 | 26.9 | 15 | 15.8 | 18 | 21.2 | 25 | 29.2 | 20 | 20.6 | 22.4 | 25 | 28.3 | 32 |
| gB2 | 11.2 | 10 | 11.2 | 14.1 | 18 | 22.4 | 15.8 | 15 | 15.8 | 18 | 21.2 | 25 | 20.6 | 20 | 20.6 | 22.4 | 25 | 28.3 |
| gB3 | 14.1 | 11.2 | 10 | 11.2 | 14.1 | 18 | 18 | 15.8 | 15 | 15.8 | 18 | 21.2 | 22.4 | 20.6 | 20 | 20.6 | 22.4 | 25 |
| gB4 | 18 | 14.1 | 11.2 | 10 | 11.2 | 14.1 | 21.2 | 18 | 15.8 | 15 | 15.8 | 18 | 25 | 22.4 | 20.6 | 20 | 20.6 | 22.4 |
| gB5 | 22.4 | 18 | 14.1 | 11.2 | 10 | 11.2 | 25 | 21.2 | 18 | 15.8 | 15 | 15.8 | 28.3 | 25 | 22.4 | 20.6 | 20 | 20.6 |
| gB6 | 26.9 | 22.4 | 18 | 14.1 | 11.2 | 10 | 29.2 | 25 | 21.2 | 18 | 15.8 | 15 | 32 | 28.3 | 25 | 22.4 | 20.6 | 20 |
| gC1 | 5 | 7.1 | 11.2 | 15.8 | 20.6 | 25.5 | 10 | 11.2 | 14.1 | 18 | 22.4 | 26.9 | 15 | 15.8 | 18 | 21.2 | 25 | 29.2 |
| gC2 | 7.1 | 5 | 7.1 | 11.2 | 15.8 | 20.6 | 11.2 | 10 | 11.2 | 14.1 | 18 | 22.4 | 15.8 | 15 | 15.8 | 18 | 21.2 | 25 |
| gC3 | 11.2 | 7.1 | 5 | 7.1 | 11.2 | 15.8 | 14.1 | 11.2 | 10 | 11.2 | 14.1 | 18 | 18 | 15.8 | 15 | 15.8 | 18 | 21.2 |
| gC4 | 15.8 | 11.2 | 7.1 | 5 | 7.1 | 11.2 | 18 | 14.1 | 11.2 | 10 | 11.2 | 14.1 | 21.2 | 18 | 15.8 | 15 | 15.8 | 18 |
| gC5 | 20.6 | 15.8 | 11.2 | 7.1 | 5 | 7.1 | 22.4 | 18 | 14.1 | 11.2 | 10 | 11.2 | 25 | 21.2 | 18 | 15.8 | 15 | 15.8 |
| gC6 | 25.5 | 20.6 | 15.8 | 11.2 | 7.1 | 5 | 26.9 | 22.4 | 18 | 14.1 | 11.2 | 10 | 29.2 | 25 | 21.2 | 18 | 15.8 | 15 |
| gD1 | 0 | 5 | 10 | 15 | 20 | 25 | 5 | 7.1 | 11.2 | 15.8 | 20.6 | 25.5 | 10 | 11.2 | 14.1 | 18 | 22.4 | 26.9 |
| gD2 | 5 | 0 | 5 | 10 | 15 | 20 | 7.1 | 5 | 7.1 | 11.2 | 15.8 | 20.6 | 11.2 | 10 | 11.2 | 14.1 | 18 | 22.4 |
| gD3 | 10 | 5 | 0 | 5 | 10 | 15 | 11.2 | 7.1 | 5 | 7.1 | 11.2 | 15.8 | 14.1 | 11.2 | 10 | 11.2 | 14.1 | 18 |
| gD4 | 15 | 10 | 5 | 0 | 5 | 10 | 15.8 | 11.2 | 7.1 | 5 | 7.1 | 11.2 | 18 | 14.1 | 11.2 | 10 | 11.2 | 14.1 |
| gD5 | 20 | 15 | 10 | 5 | 0 | 5 | 20.6 | 15.8 | 11.2 | 7.1 | 5 | 7.1 | 22.4 | 18 | 14.1 | 11.2 | 10 | 11.2 |
| gD6 | 25 | 20 | 15 | 10 | 5 | 0 | 25.5 | 20.6 | 15.8 | 11.2 | 7.1 | 5 | 26.9 | 22.4 | 18 | 14.1 | 11.2 | 10 |
| gE1 | 5 | 7.1 | 11.2 | 15.8 | 20.6 | 25.5 | 0 | 5 | 10 | 15 | 20 | 25 | 5 | 7.1 | 11.2 | 15.8 | 20.6 | 25.5 |
| gE2 | 7.1 | 5 | 7.1 | 11.2 | 15.8 | 20.6 | 5 | 0 | 5 | 10 | 15 | 20 | 7.1 | 5 | 7.1 | 11.2 | 15.8 | 20.6 |
| gE3 | 11.2 | 7.1 | 5 | 7.1 | 11.2 | 15.8 | 10 | 5 | 0 | 5 | 10 | 15 | 11.2 | 7.1 | 5 | 7.1 | 11.2 | 15.8 |
| gE4 | 15.8 | 11.2 | 7.1 | 5 | 7.1 | 11.2 | 15 | 10 | 5 | 0 | 5 | 10 | 15.8 | 11.2 | 7.1 | 5 | 7.1 | 11.2 |
| gE5 | 20.6 | 15.8 | 11.2 | 7.1 | 5 | 7.1 | 20 | 15 | 10 | 5 | 0 | 5 | 20.6 | 15.8 | 11.2 | 7.1 | 5 | 7.1 |
| gE6 | 25.5 | 20.6 | 15.8 | 11.2 | 7.1 | 5 | 25 | 20 | 15 | 10 | 5 | 0 | 25.5 | 20.6 | 15.8 | 11.2 | 7.1 | 5 |
| gF1 | 10 | 11.2 | 14.1 | 18 | 22.4 | 26.9 | 5 | 7.1 | 11.2 | 15.8 | 20.6 | 25.5 | 0 | 5 | 10 | 15 | 20 | 25 |
| gF2 | 11.2 | 10 | 11.2 | 14.1 | 18 | 22.4 | 7.1 | 5 | 7.1 | 11.2 | 15.8 | 20.6 | 5 | 0 | 5 | 10 | 15 | 20 |
| gF3 | 14.1 | 11.2 | 10 | 11.2 | 14.1 | 18 | 11.2 | 7.1 | 5 | 7.1 | 11.2 | 15.8 | 10 | 5 | 0 | 5 | 10 | 15 |
| gF4 | 18 | 14.1 | 11.2 | 10 | 11.2 | 14.1 | 15.8 | 11.2 | 7.1 | 5 | 7.1 | 11.2 | 15 | 10 | 5 | 0 | 5 | 10 |
| gF5 | 22.4 | 18 | 14.1 | 11.2 | 10 | 11.2 | 20.6 | 15.8 | 11.2 | 7.1 | 5 | 7.1 | 20 | 15 | 10 | 5 | 0 | 5 |
| gF6 | 26.9 | 22.4 | 18 | 14.1 | 11.2 | 10 | 25.5 | 20.6 | 15.8 | 11.2 | 7.1 | 5 | 25 | 20 | 15 | 10 | 5 | 0 |

## B. EXAMPLE TWO: SHORT-HANDED SCENARIO

### Sets

| $P_{VIS}$ | $P_{SECRET}$ | $P_{FLEX}$ | $P_{BASE}$ | $P_{TETH}$ | $PP_{TETH}$ | $M_{ps}$ |
|---|---|---|---|---|---|---|
| Surf1 | (empty) | SSN1 | (empty) | (empty) | (empty) | {Surf1.m1.active,…,Surf1.m214.active} |
| | | | | | | {SSN1.m1.active,…,SSN1.m214.active} |
| | | | | | | {SSN1.m1.passive,…,SSN1.m214.passive} |

These sets are identical to Example One:
***A***
$G_{pm}$

### Data

$teth\_range_{p''}$
(empty)

These data are identical to Example One:
$dist_q$
$range_{m'm''}$

$trans_p$
Surf1 0.5
SSN1 0.7

$r_{psq}$

| | Surf1.active | SSN1.passive | SSN1.active |
|---|---|---|---|
| gA1 | 0.42 | 0.45 | 0.63 |
| gA2 | 0.42 | 0.45 | 0.63 |
| gA3 | 0.56 | 0.6 | 0.84 |
| gA4 | 0.7 | 0.75 | 1.05 |
| gA5 | 0.91 | 0.97 | 1.36 |
| gA6 | 0.95 | 1.02 | 1.43 |
| gB1 | 0.42 | 0.45 | 0.63 |
| gB2 | 0.49 | 0.53 | 0.74 |
| gB3 | 0.56 | 0.6 | 0.84 |
| gB4 | 0.7 | 0.75 | 1.05 |
| gB5 | 0.88 | 0.94 | 1.32 |
| gB6 | 1.03 | 1.11 | 1.55 |
| gC1 | 0.56 | 0.6 | 0.84 |
| gC2 | 0.63 | 0.68 | 0.95 |
| gC3 | 0.7 | 0.75 | 1.05 |
| gC4 | 0.84 | 0.9 | 1.26 |
| gC5 | 0.91 | 0.98 | 1.37 |
| gC6 | 0.97 | 1.03 | 1.45 |
| gD1 | 0.91 | 0.98 | 1.37 |
| gD2 | 1.01 | 1.08 | 1.52 |
| gD3 | 0.77 | 0.83 | 1.16 |
| gD4 | 0.63 | 0.68 | 0.95 |
| gD5 | 0.7 | 0.75 | 1.05 |
| gD6 | 0.7 | 0.75 | 1.05 |
| gE1 | 0.99 | 1.06 | 1.48 |
| gE2 | 1.03 | 1.1 | 1.55 |
| gE3 | 0.63 | 0.68 | 0.95 |
| gE4 | 0.49 | 0.53 | 0.74 |
| gE5 | 0.56 | 0.6 | 0.84 |
| gE6 | 0.56 | 0.6 | 0.84 |
| gF1 | 0.95 | 1.02 | 1.43 |
| gF2 | 0.88 | 0.94 | 1.31 |
| gF3 | 0.56 | 0.6 | 0.84 |
| gF4 | 0.42 | 0.45 | 0.63 |
| gF5 | 0.42 | 0.45 | 0.63 |
| gF6 | 0.42 | 0.45 | 0.63 |

## C. EXAMPLE THREE: OCEAN INFLUENCE SCENARIO

# Sets

All sets are identical to Example One.

# Data

These data are identical to Example One:

***dist*** $_{q}$

***teth_range*** $_{p''}$

***trans*** $_{p}$

***range*** $_{m'm''}$

***r*** $_{psq}$

| | Surf1.active | Surf2.active | Surf3.active | Helo1.active | P31.active | SSN1.passive | SSN1.active |
|---|---|---|---|---|---|---|---|
| gA1 | 0.7 | 0.7 | 0.7 | 0.84 | 0.91 | 0.75 | 1.05 |
| gA2 | 1.4 | 1.4 | 1.4 | 1.68 | 1.82 | 1.5 | 2.1 |
| gA3 | 0.49 | 0.49 | 0.49 | 0.59 | 0.64 | 0.53 | 0.74 |
| gA4 | 0.28 | 0.28 | 0.28 | 0.34 | 0.36 | 0.3 | 0.42 |
| gA5 | 0.28 | 0.28 | 0.28 | 0.34 | 0.36 | 0.3 | 0.42 |
| gA6 | 0.7 | 0.7 | 0.7 | 0.84 | 0.91 | 0.75 | 1.05 |
| gB1 | 0.84 | 0.84 | 0.84 | 1.01 | 1.09 | 0.9 | 1.26 |
| gB2 | 0.77 | 0.77 | 0.77 | 0.92 | 1 | 0.83 | 1.16 |
| gB3 | 0.35 | 0.35 | 0.35 | 0.42 | 0.46 | 0.38 | 0.53 |
| gB4 | 0.24 | 0.24 | 0.24 | 0.29 | 0.31 | 0.26 | 0.36 |
| gB5 | 0.28 | 0.28 | 0.28 | 0.34 | 0.36 | 0.3 | 0.42 |
| gB6 | 1.12 | 1.12 | 1.12 | 1.34 | 1.46 | 1.2 | 1.68 |
| gC1 | 0.84 | 0.84 | 0.84 | 1.01 | 1.09 | 0.9 | 1.26 |
| gC2 | 0.63 | 0.63 | 0.63 | 0.76 | 0.82 | 0.68 | 0.95 |
| gC3 | 0.49 | 0.49 | 0.49 | 0.59 | 0.64 | 0.53 | 0.74 |
| gC4 | 0.28 | 0.28 | 0.28 | 0.34 | 0.36 | 0.3 | 0.42 |
| gC5 | 0.14 | 0.14 | 0.14 | 0.17 | 0.18 | 0.15 | 0.21 |
| gC6 | 1.12 | 1.12 | 1.12 | 1.34 | 1.46 | 1.2 | 1.68 |
| gD1 | 0.7 | 0.7 | 0.7 | 0.84 | 0.91 | 0.75 | 1.05 |
| gD2 | 0.42 | 0.42 | 0.42 | 0.5 | 0.55 | 0.45 | 0.63 |
| gD3 | 0.28 | 0.28 | 0.28 | 0.34 | 0.36 | 0.3 | 0.42 |
| gD4 | 0.28 | 0.28 | 0.28 | 0.34 | 0.36 | 0.3 | 0.42 |
| gD5 | 0.35 | 0.35 | 0.35 | 0.42 | 0.46 | 0.38 | 0.53 |
| gD6 | 1.4 | 1.4 | 1.4 | 1.68 | 1.82 | 1.5 | 2.1 |
| gE1 | 0.7 | 0.7 | 0.7 | 0.84 | 0.91 | 0.75 | 1.05 |
| gE2 | 0.39 | 0.39 | 0.39 | 0.47 | 0.51 | 0.42 | 0.59 |
| gE3 | 0.28 | 0.28 | 0.28 | 0.34 | 0.36 | 0.3 | 0.42 |
| gE4 | 0.28 | 0.28 | 0.28 | 0.34 | 0.36 | 0.3 | 0.42 |
| gE5 | 0.7 | 0.7 | 0.7 | 0.84 | 0.91 | 0.75 | 1.05 |
| gE6 | 1.4 | 1.4 | 1.4 | 1.68 | 1.82 | 1.5 | 2.1 |
| gF1 | 0.7 | 0.7 | 0.7 | 0.84 | 0.91 | 0.75 | 1.05 |
| gF2 | 0.49 | 0.49 | 0.49 | 0.59 | 0.64 | 0.53 | 0.74 |
| gF3 | 0.28 | 0.28 | 0.28 | 0.34 | 0.36 | 0.3 | 0.42 |
| gF4 | 0.28 | 0.28 | 0.28 | 0.34 | 0.36 | 0.3 | 0.42 |
| gF5 | 0.49 | 0.49 | 0.49 | 0.59 | 0.64 | 0.53 | 0.74 |
| gF6 | 1.4 | 1.4 | 1.4 | 1.68 | 1.82 | 1.5 | 2.1 |

## D. EXAMPLE FOUR: MULTIPLE HVU SCENARIO

### Sets

| ***P*** $_{VIS}$ | ***P*** $_{SECRET}$ | ***P*** $_{FLEX}$ | ***P*** $_{BASE}$ | ***P*** $_{TETH}$ | ***PP*** $_{TETH}$ |
|---|---|---|---|---|---|
| Surf1 | SSN2 | SSN1 | Surf1 | Helo1 | Surf1.Helo1 |
| Surf2 | | | Surf2 | Helo2 | Surf2.Helo2 |
| Surf3 | | | | | |
| Helo1 | | | | | |
| Helo2 | | | | | |
| P31 | | | | | |

***G***

$g^{+}$
gA1
gA2
gA3
gA4
gA5
gA6
gA7
gA8
gB1
gB2
gB3
gB4
gB5
gB6
gB7
gB8
gC1
gC2
gC3
gC4
gC5
gC6 (protected)
gC7
gC8
gD1
gD2
gD3
gD4
gD5
gD6
gD7
gD8
gE1
gE2
gE3
gE4
gE5
gE6
gE7
gE8
gF1
gF2
gF3 (protected)
gF4
gF5
gF6
gF7
gF8
gG1
gG2
gG3
gG4
gG5
gG6
gG7
gG8
gH1
gH2
gH3
gH4
gH5
gH6
gH7
gH8
$g^{-}$

| ***M*** | ***M*** $_{ps}$ |
|---|---|
| {m1,…,m468} | {Surf1.m1.active,…,Surf1.m468.active} |
| | {Surf2.m1.active,…,Surf2.m468.active} |
| | {Surf3.m1.active,…,Surf3.m468.active} |
| | {Helo1.m1.active,…,Helo1.m468.active} |
| | {Helo2.m1.active,…,Helo2.m468.active} |
| | {P31.m1.active,…,P31.m468.active} |
| | {SSN1.m1.active,…,SSN1.m100.active} |
| | {SSN1.m1.passive,…,SSN1.m100.passive} |
| | {SSN2.m1.passive,…,SSN2.m210.passive} |

***A***

g+.gA1
g+.gA2
g+.gA3
g+.gA4
g+.gA5
g+.gA6
g+.gA7
g+.gA8
g+.gB1
g+.gB8
g+.gC1
g+.gC8
g+.gD1
g+.gD8
g+.gE1
g+.gE8
g+.gF1
g+.gF8
g+.gG1
g+.gG8
g+.gH1
g+.gH2
g+.gH3
g+.gH4
g+.gH5
g+.gH6
g+.gH7
g+.gH8
gA1.gA2
gA1.gB1
gA1.gB2
gA2.gA1
gA2.gA3
gA2.gB1
gA2.gB2
gA2.gB3
gA3.gA2
gA3.gA4
gA3.gB2
gA3.gB3
gA3.gB4
gA4.gA3
gA4.gA5
gA4.gB3
gA4.gB4
gA4.gB5
gA5.gA4
gA5.gA6
gA5.gB4
gA5.gB5
gA5.gB6
gA6.gA5
gA6.gA7
gA6.gB5
gA6.gB6
gA6.gB7
gA7.gA6
gA7.gA8
gA7.gB6
gA7.gB7
gA7.gB8
gA8.gA7
gA8.gB7
gA8.gB8
gB1.gA1
gB1.gA2
gB1.gB2
gB1.gC1
gB1.gC2
gB2.gA1
gB2.gA2
gB2.gA3
gB2.gB1
gB2.gB3
gB2.gC1
gB2.gC2
gB2.gC3
gB3.gA2
gB3.gA3
gB3.gA4
gB3.gB2
gB3.gB4
gB3.gC2
gB3.gC3
gB3.gC4
gB4.gA3
gB4.gA4
gB4.gA5
gB4.gB3
gB4.gB5
gB4.gC3
gB4.gC4
gB4.gC5
gB5.gA4
gB5.gA5
gB5.gA6
gB5.gB4
gB5.gB6
gB5.gC4
gB5.gC5
gB5.gC6
gB5.g-
gB6.gA5
gB6.gA6
gB6.gA7
gB6.gB5
gB6.gB7
gB6.gC5
gB6.gC6
gB6.gC7
gB6.g-
gB7.gA6
gB7.gA7
gB7.gA8
gB7.gB6
gB7.gB8
gB7.gC6
gB7.gC7
gB7.gC8
gB7.g-
gB8.gA7
gB8.gA8
gB8.gB7
gB8.gC7
gB8.gC8
gC1.gB1
gC1.gB2
gC1.gC2
gC1.gD1
gC1.gD2
gC2.gB1
gC2.gB2
gC2.gB3
gC2.gC1
gC2.gC3
gC2.gD1
gC2.gD2
gC2.gD3
gC3.gB2
gC3.gB3
gC3.gB4
gC3.gC2
gC3.gC4
gC3.gD2
gC3.gD3
gC3.gD4
gC4.gB3
gC4.gB4
gC4.gB5
gC4.gC3
gC4.gC5
gC4.gD3
gC4.gD4
gC4.gD5
gC5.gB4
gC5.gB5
gC5.gB6
gC5.gC4
gC5.gC6
gC5.gD4
gC5.gD5
gC5.gD6
gC5.g-
gC6.gB5
gC6.gB6
gC6.gB7
gC6.gC5
gC6.gC7
gC6.gD5
gC6.gD6
gC6.gD7
gC7.gB6
gC7.gB7
gC7.gB8
gC7.gC6
gC7.gC8
gC7.gD6
gC7.gD7
gC7.gD8
gC7.g-
gC8.gB7
gC8.gB8
gC8.gC7
gC8.gD7
gC8.gD8
gD1.gC1
gD1.gC2
gD1.gD2
gD1.gE1
gD1.gE2
gD2.gC1
gD2.gC2
gD2.gC3
gD2.gD1
gD2.gD3
gD2.gE1
gD2.gE2
gD2.gE3
gD3.gC2
gD3.gC3
gD3.gC4
gD3.gD2
gD3.gD4
gD3.gE2
gD3.gE3
gD3.gE4
gD4.gC3
gD4.gC4
gD4.gC5
gD4.gD3
gD4.gD5
gD4.gE3
gD4.gE4
gD4.gE5
gD5.gC4
gD5.gC5
gD5.gC6
gD5.gD4
gD5.gD6
gD5.gE4
gD5.gE5
gD5.gE6
gD5.g-
gD6.gC5
gD6.gC6
gD6.gC7
gD6.gD5
gD6.gD7
gD6.gE5
gD6.gE6
gD6.gE7
gD6.g-
gD7.gC6
gD7.gC7
gD7.gC8
gD7.gD6
gD7.gD8
gD7.gE6
gD7.gE7
gD7.gE8
gD7.g-
gD8.gC7
gD8.gC8
gD8.gD7
gD8.gE7
gD8.gE8
gE1.gD1
gE1.gD2
gE1.gE2
gE1.gF1
gE1.gF2
gE2.gD1
gE2.gD2
gE2.gD3
gE2.gE1
gE2.gE3
gE2.gF1
gE2.gF2
gE2.gF3
gE2.g-
gE3.gD2
gE3.gD3
gE3.gD4
gE3.gE2
gE3.gE4
gE3.gF2
gE3.gF3
gE3.gF4
gE3.g-
gE4.gD3
gE4.gD4
gE4.gD5
gE4.gE3
gE4.gE5
gE4.gF3
gE4.gF4
gE4.gF5
gE4.g-
gE5.gD4
gE5.gD5
gE5.gD6
gE5.gE4
gE5.gE6
gE5.gF4
gE5.gF5
gE5.gF6
gE6.gD5
gE6.gD6
gE6.gD7
gE6.gE5
gE6.gE7
gE6.gF5
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gH8.gH7

***G**$_{pm}$* ***(for all p except SSN1 and SSN2)***

| | | | | | |
|---|---|---|---|---|---|
| p.m1.gA1 | p.m51.gG3 | p.m83.gC3 | p.m108.gF4 | p.m133.gB6 | p.m158.gF3 |
| p.m2.gA2 | p.m52.gG4 | p.m83.gD3 | p.m108.gG4 | p.m133.gB7 | p.m158.gF4 |
| p.m3.gA3 | p.m53.gG5 | p.m84.gC4 | p.m109.gF5 | p.m134.gB7 | p.m159.gF4 |
| p.m4.gA4 | p.m54.gG6 | p.m84.gD4 | p.m109.gG5 | p.m134.gB8 | p.m159.gF5 |
| p.m5.gA5 | p.m55.gG7 | p.m85.gC5 | p.m110.gF6 | p.m135.gC1 | p.m160.gF5 |
| p.m6.gA6 | p.m56.gG8 | p.m85.gD5 | p.m110.gG6 | p.m135.gC2 | p.m160.gF6 |
| p.m7.gA7 | p.m57.gH1 | p.m86.gC6 | p.m111.gF7 | p.m136.gC2 | p.m161.gF6 |
| p.m8.gA8 | p.m58.gH2 | p.m86.gD6 | p.m111.gG7 | p.m136.gC3 | p.m161.gF7 |
| p.m9.gB1 | p.m59.gH3 | p.m87.gC7 | p.m112.gF8 | p.m137.gC3 | p.m162.gF7 |
| p.m10.gB2 | p.m60.gH4 | p.m87.gD7 | p.m112.gG8 | p.m137.gC4 | p.m162.gF8 |
| p.m11.gB3 | p.m61.gH5 | p.m88.gC8 | p.m113.gG1 | p.m138.gC4 | p.m163.gG1 |
| p.m12.gB4 | p.m62.gH6 | p.m88.gD8 | p.m113.gH1 | p.m138.gC5 | p.m163.gG2 |
| p.m13.gB5 | p.m63.gH7 | p.m89.gD1 | p.m114.gG2 | p.m139.gC5 | p.m164.gG2 |
| p.m14.gB6 | p.m64.gH8 | p.m89.gE1 | p.m114.gH2 | p.m139.gC6 | p.m164.gG3 |
| p.m15.gB7 | p.m65.gA1 | p.m90.gD2 | p.m115.gG3 | p.m140.gC6 | p.m165.gG3 |
| p.m16.gB8 | p.m65.gB1 | p.m90.gE2 | p.m115.gH3 | p.m140.gC7 | p.m165.gG4 |
| p.m17.gC1 | p.m66.gA2 | p.m91.gD3 | p.m116.gG4 | p.m141.gC7 | p.m166.gG4 |
| p.m18.gC2 | p.m66.gB2 | p.m91.gE3 | p.m116.gH4 | p.m141.gC8 | p.m166.gG5 |
| p.m19.gC3 | p.m67.gA3 | p.m92.gD4 | p.m117.gG5 | p.m142.gD1 | p.m167.gG5 |
| p.m20.gC4 | p.m67.gB3 | p.m92.gE4 | p.m117.gH5 | p.m142.gD2 | p.m167.gG6 |
| p.m21.gC5 | p.m68.gA4 | p.m93.gD5 | p.m118.gG6 | p.m143.gD2 | p.m168.gG6 |
| p.m22.gC6 | p.m68.gB4 | p.m93.gE5 | p.m118.gH6 | p.m143.gD3 | p.m168.gG7 |
| p.m23.gC7 | p.m69.gA5 | p.m94.gD6 | p.m119.gG7 | p.m144.gD3 | p.m169.gG7 |
| p.m24.gC8 | p.m69.gB5 | p.m94.gE6 | p.m119.gH7 | p.m144.gD4 | p.m169.gG8 |
| p.m25.gD1 | p.m70.gA6 | p.m95.gD7 | p.m120.gG8 | p.m145.gD4 | p.m170.gH1 |
| p.m26.gD2 | p.m70.gB6 | p.m95.gE7 | p.m120.gH8 | p.m145.gD5 | p.m170.gH2 |
| p.m27.gD3 | p.m71.gA7 | p.m96.gD8 | p.m121.gA1 | p.m146.gD5 | p.m171.gH2 |
| p.m28.gD4 | p.m71.gB7 | p.m96.gE8 | p.m121.gA2 | p.m146.gD6 | p.m171.gH3 |
| p.m29.gD5 | p.m72.gA8 | p.m97.gE1 | p.m122.gA2 | p.m147.gD6 | p.m172.gH3 |
| p.m30.gD6 | p.m72.gB8 | p.m97.gF1 | p.m122.gA3 | p.m147.gD7 | p.m172.gH4 |
| p.m31.gD7 | p.m73.gB1 | p.m98.gE2 | p.m123.gA3 | p.m148.gD7 | p.m173.gH4 |
| p.m32.gD8 | p.m73.gC1 | p.m98.gF2 | p.m123.gA4 | p.m148.gD8 | p.m173.gH5 |
| p.m33.gE1 | p.m74.gB2 | p.m99.gE3 | p.m124.gA4 | p.m149.gE1 | p.m174.gH5 |
| p.m34.gE2 | p.m74.gC2 | p.m99.gF3 | p.m124.gA5 | p.m149.gE2 | p.m174.gH6 |
| p.m35.gE3 | p.m75.gB3 | p.m100.gE4 | p.m125.gA5 | p.m150.gE2 | p.m175.gH6 |
| p.m36.gE4 | p.m75.gC3 | p.m100.gF4 | p.m125.gA6 | p.m150.gE3 | p.m175.gH7 |
| p.m37.gE5 | p.m76.gB4 | p.m101.gE5 | p.m126.gA6 | p.m151.gE3 | p.m176.gH7 |
| p.m38.gE6 | p.m76.gC4 | p.m101.gF5 | p.m126.gA7 | p.m151.gE4 | p.m176.gH8 |
| p.m39.gE7 | p.m77.gB5 | p.m102.gE6 | p.m127.gA7 | p.m152.gE4 | p.m177.gA1 |
| p.m40.gE8 | p.m77.gC5 | p.m102.gF6 | p.m127.gA8 | p.m152.gE5 | p.m177.gB1 |
| p.m41.gF1 | p.m78.gB6 | p.m103.gE7 | p.m128.gB1 | p.m153.gE5 | p.m177.gC1 |
| p.m42.gF2 | p.m78.gC6 | p.m103.gF7 | p.m128.gB2 | p.m153.gE6 | p.m178.gA2 |
| p.m43.gF3 | p.m79.gB7 | p.m104.gE8 | p.m129.gB2 | p.m154.gE6 | p.m178.gB2 |
| p.m44.gF4 | p.m79.gC7 | p.m104.gF8 | p.m129.gB3 | p.m154.gE7 | p.m178.gC2 |
| p.m45.gF5 | p.m80.gB8 | p.m105.gF1 | p.m130.gB3 | p.m155.gE7 | p.m179.gA3 |
| p.m46.gF6 | p.m80.gC8 | p.m105.gG1 | p.m130.gB4 | p.m155.gE8 | p.m179.gB3 |
| p.m47.gF7 | p.m81.gC1 | p.m106.gF2 | p.m131.gB4 | p.m156.gF1 | p.m179.gC3 |
| p.m48.gF8 | p.m81.gD1 | p.m106.gG2 | p.m131.gB5 | p.m156.gF2 | p.m180.gA4 |
| p.m49.gG1 | p.m82.gC2 | p.m107.gF3 | p.m132.gB5 | p.m157.gF2 | p.m180.gB4 |
| p.m50.gG2 | p.m82.gD2 | p.m107.gG3 | p.m132.gB6 | p.m157.gF3 | p.m180.gC4 |

***$G_{pm}$ (for all p except SSN1 and SSN2) (continued)***

| | | | | | |
|---|---|---|---|---|---|
| p.m181.gA5 | p.m197.gE5 | p.m214.gF6 | p.m231.gB1 | p.m247.gD7 | p.m264.gG5 |
| p.m181.gB5 | p.m198.gC6 | p.m214.gG6 | p.m231.gB2 | p.m248.gD6 | p.m264.gG6 |
| p.m181.gC5 | p.m198.gD6 | p.m215.gE7 | p.m231.gB3 | p.m248.gD7 | p.m265.gG5 |
| p.m182.gA6 | p.m198.gE6 | p.m215.gF7 | p.m232.gB2 | p.m248.gD8 | p.m265.gG6 |
| p.m182.gB6 | p.m199.gC7 | p.m215.gG7 | p.m232.gB3 | p.m249.gE1 | p.m265.gG7 |
| p.m182.gC6 | p.m199.gD7 | p.m216.gE8 | p.m232.gB4 | p.m249.gE2 | p.m266.gG6 |
| p.m183.gA7 | p.m199.gE7 | p.m216.gF8 | p.m233.gB3 | p.m249.gE3 | p.m266.gG7 |
| p.m183.gB7 | p.m200.gC8 | p.m216.gG8 | p.m233.gB4 | p.m250.gE2 | p.m266.gG8 |
| p.m183.gC7 | p.m200.gD8 | p.m217.gF1 | p.m233.gB5 | p.m250.gE3 | p.m267.gH1 |
| p.m184.gA8 | p.m200.gE8 | p.m217.gG1 | p.m234.gB4 | p.m250.gE4 | p.m267.gH2 |
| p.m184.gB8 | p.m201.gD1 | p.m217.gH1 | p.m234.gB5 | p.m251.gE3 | p.m267.gH3 |
| p.m184.gC8 | p.m201.gE1 | p.m218.gF2 | p.m234.gB6 | p.m251.gE4 | p.m268.gH2 |
| p.m185.gB1 | p.m201.gF1 | p.m218.gG2 | p.m235.gB5 | p.m251.gE5 | p.m268.gH3 |
| p.m185.gC1 | p.m202.gD2 | p.m218.gH2 | p.m235.gB6 | p.m252.gE4 | p.m268.gH4 |
| p.m185.gD1 | p.m202.gE2 | p.m219.gF3 | p.m235.gB7 | p.m252.gE5 | p.m269.gH3 |
| p.m186.gB2 | p.m202.gF2 | p.m219.gG3 | p.m236.gB6 | p.m252.gE6 | p.m269.gH4 |
| p.m186.gC2 | p.m203.gD3 | p.m219.gH3 | p.m236.gB7 | p.m253.gE5 | p.m269.gH5 |
| p.m186.gD2 | p.m203.gE3 | p.m220.gF4 | p.m236.gB8 | p.m253.gE6 | p.m270.gH4 |
| p.m187.gB3 | p.m203.gF3 | p.m220.gG4 | p.m237.gC1 | p.m253.gE7 | p.m270.gH5 |
| p.m187.gC3 | p.m204.gD4 | p.m220.gH4 | p.m237.gC2 | p.m254.gE6 | p.m270.gH6 |
| p.m187.gD3 | p.m204.gE4 | p.m221.gF5 | p.m237.gC3 | p.m254.gE7 | p.m271.gH5 |
| p.m188.gB4 | p.m204.gF4 | p.m221.gG5 | p.m238.gC2 | p.m254.gE8 | p.m271.gH6 |
| p.m188.gC4 | p.m205.gD5 | p.m221.gH5 | p.m238.gC3 | p.m255.gF1 | p.m271.gH7 |
| p.m188.gD4 | p.m205.gE5 | p.m222.gF6 | p.m238.gC4 | p.m255.gF2 | p.m272.gH6 |
| p.m189.gB5 | p.m205.gF5 | p.m222.gG6 | p.m239.gC3 | p.m255.gF3 | p.m272.gH7 |
| p.m189.gC5 | p.m206.gD6 | p.m222.gH6 | p.m239.gC4 | p.m256.gF2 | p.m272.gH8 |
| p.m189.gD5 | p.m206.gE6 | p.m223.gF7 | p.m239.gC5 | p.m256.gF3 | p.m273.gA1 |
| p.m190.gB6 | p.m206.gF6 | p.m223.gG7 | p.m240.gC4 | p.m256.gF4 | p.m273.gA2 |
| p.m190.gC6 | p.m207.gD7 | p.m223.gH7 | p.m240.gC5 | p.m257.gF3 | p.m273.gB1 |
| p.m190.gD6 | p.m207.gE7 | p.m224.gF8 | p.m240.gC6 | p.m257.gF4 | p.m274.gA2 |
| p.m191.gB7 | p.m207.gF7 | p.m224.gG8 | p.m241.gC5 | p.m257.gF5 | p.m274.gA3 |
| p.m191.gC7 | p.m208.gD8 | p.m224.gH8 | p.m241.gC6 | p.m258.gF4 | p.m274.gB2 |
| p.m191.gD7 | p.m208.gE8 | p.m225.gA1 | p.m241.gC7 | p.m258.gF5 | p.m275.gA3 |
| p.m192.gB8 | p.m208.gF8 | p.m225.gA2 | p.m242.gC6 | p.m258.gF6 | p.m275.gA4 |
| p.m192.gC8 | p.m209.gE1 | p.m225.gA3 | p.m242.gC7 | p.m259.gF5 | p.m275.gB3 |
| p.m192.gD8 | p.m209.gF1 | p.m226.gA2 | p.m242.gC8 | p.m259.gF6 | p.m276.gA4 |
| p.m193.gC1 | p.m209.gG1 | p.m226.gA3 | p.m243.gD1 | p.m259.gF7 | p.m276.gA5 |
| p.m193.gD1 | p.m210.gE2 | p.m226.gA4 | p.m243.gD2 | p.m260.gF6 | p.m276.gB4 |
| p.m193.gE1 | p.m210.gF2 | p.m227.gA3 | p.m243.gD3 | p.m260.gF7 | p.m277.gA5 |
| p.m194.gC2 | p.m210.gG2 | p.m227.gA4 | p.m244.gD2 | p.m260.gF8 | p.m277.gA6 |
| p.m194.gD2 | p.m211.gE3 | p.m227.gA5 | p.m244.gD3 | p.m261.gG1 | p.m277.gB5 |
| p.m194.gE2 | p.m211.gF3 | p.m228.gA4 | p.m244.gD4 | p.m261.gG2 | p.m278.gA6 |
| p.m195.gC3 | p.m211.gG3 | p.m228.gA5 | p.m245.gD3 | p.m261.gG3 | p.m278.gA7 |
| p.m195.gD3 | p.m212.gE4 | p.m228.gA6 | p.m245.gD4 | p.m262.gG2 | p.m278.gB6 |
| p.m195.gE3 | p.m212.gF4 | p.m229.gA5 | p.m245.gD5 | p.m262.gG3 | p.m279.gA7 |
| p.m196.gC4 | p.m212.gG4 | p.m229.gA6 | p.m246.gD4 | p.m262.gG4 | p.m279.gA8 |
| p.m196.gD4 | p.m213.gE5 | p.m229.gA7 | p.m246.gD5 | p.m263.gG3 | p.m279.gB7 |
| p.m196.gE4 | p.m213.gF5 | p.m230.gA6 | p.m246.gD6 | p.m263.gG4 | p.m280.gB1 |
| p.m197.gC5 | p.m213.gG5 | p.m230.gA7 | p.m247.gD5 | p.m263.gG5 | p.m280.gB2 |
| p.m197.gD5 | p.m214.gE6 | p.m230.gA8 | p.m247.gD6 | p.m264.gG4 | p.m280.gC1 |

***$G_{pm}$ (for all p except SSN1 and SSN2) (continued)***

| | | | | | |
|---|---|---|---|---|---|
| p.m281.gB2 | p.m297.gE4 | p.m314.gF8 | p.m331.gB3 | p.m347.gE6 | p.m364.gH1 |
| p.m281.gB3 | p.m298.gD5 | p.m314.gG7 | p.m331.gC3 | p.m348.gD6 | p.m364.gH2 |
| p.m281.gC2 | p.m298.gD6 | p.m315.gG1 | p.m331.gC4 | p.m348.gE6 | p.m365.gG2 |
| p.m282.gB3 | p.m298.gE5 | p.m315.gG2 | p.m332.gB4 | p.m348.gE7 | p.m365.gH2 |
| p.m282.gB4 | p.m299.gD6 | p.m315.gH1 | p.m332.gC4 | p.m349.gD7 | p.m365.gH3 |
| p.m282.gC3 | p.m299.gD7 | p.m316.gG2 | p.m332.gC5 | p.m349.gE7 | p.m366.gG3 |
| p.m283.gB4 | p.m299.gE6 | p.m316.gG3 | p.m333.gB5 | p.m349.gE8 | p.m366.gH3 |
| p.m283.gB5 | p.m300.gD7 | p.m316.gH2 | p.m333.gC5 | p.m350.gE1 | p.m366.gH4 |
| p.m283.gC4 | p.m300.gD8 | p.m317.gG3 | p.m333.gC6 | p.m350.gF1 | p.m367.gG4 |
| p.m284.gB5 | p.m300.gE7 | p.m317.gG4 | p.m334.gB6 | p.m350.gF2 | p.m367.gH4 |
| p.m284.gB6 | p.m301.gE1 | p.m317.gH3 | p.m334.gC6 | p.m351.gE2 | p.m367.gH5 |
| p.m284.gC5 | p.m301.gE2 | p.m318.gG4 | p.m334.gC7 | p.m351.gF2 | p.m368.gG5 |
| p.m285.gB6 | p.m301.gF1 | p.m318.gG5 | p.m335.gB7 | p.m351.gF3 | p.m368.gH5 |
| p.m285.gB7 | p.m302.gE2 | p.m318.gH4 | p.m335.gC7 | p.m352.gE3 | p.m368.gH6 |
| p.m285.gC6 | p.m302.gE3 | p.m319.gG5 | p.m335.gC8 | p.m352.gF3 | p.m369.gG6 |
| p.m286.gB7 | p.m302.gF2 | p.m319.gG6 | p.m336.gC1 | p.m352.gF4 | p.m369.gH6 |
| p.m286.gB8 | p.m303.gE3 | p.m319.gH5 | p.m336.gD1 | p.m353.gE4 | p.m369.gH7 |
| p.m286.gC7 | p.m303.gE4 | p.m320.gG6 | p.m336.gD2 | p.m353.gF4 | p.m370.gG7 |
| p.m287.gC1 | p.m303.gF3 | p.m320.gG7 | p.m337.gC2 | p.m353.gF5 | p.m370.gH7 |
| p.m287.gC2 | p.m304.gE4 | p.m320.gH6 | p.m337.gD2 | p.m354.gE5 | p.m370.gH8 |
| p.m287.gD1 | p.m304.gE5 | p.m321.gG7 | p.m337.gD3 | p.m354.gF5 | p.m371.gB1 |
| p.m288.gC2 | p.m304.gF4 | p.m321.gG8 | p.m338.gC3 | p.m354.gF6 | p.m371.gA2 |
| p.m288.gC3 | p.m305.gE5 | p.m321.gH7 | p.m338.gD3 | p.m355.gE6 | p.m371.gB2 |
| p.m288.gD2 | p.m305.gE6 | p.m322.gA1 | p.m338.gD4 | p.m355.gF6 | p.m372.gB2 |
| p.m289.gC3 | p.m305.gF5 | p.m322.gB1 | p.m339.gC4 | p.m355.gF7 | p.m372.gA3 |
| p.m289.gC4 | p.m306.gE6 | p.m322.gB2 | p.m339.gD4 | p.m356.gE7 | p.m372.gB3 |
| p.m289.gD3 | p.m306.gE7 | p.m323.gA2 | p.m339.gD5 | p.m356.gF7 | p.m373.gB3 |
| p.m290.gC4 | p.m306.gF6 | p.m323.gB2 | p.m340.gC5 | p.m356.gF8 | p.m373.gA4 |
| p.m290.gC5 | p.m307.gE7 | p.m323.gB3 | p.m340.gD5 | p.m357.gF1 | p.m373.gB4 |
| p.m290.gD4 | p.m307.gE8 | p.m324.gA3 | p.m340.gD6 | p.m357.gG1 | p.m374.gB4 |
| p.m291.gC5 | p.m307.gF7 | p.m324.gB3 | p.m341.gC6 | p.m357.gG2 | p.m374.gA5 |
| p.m291.gC6 | p.m308.gF1 | p.m324.gB4 | p.m341.gD6 | p.m358.gF2 | p.m374.gB5 |
| p.m291.gD5 | p.m308.gF2 | p.m325.gA4 | p.m341.gD7 | p.m358.gG2 | p.m375.gB5 |
| p.m292.gC6 | p.m308.gG1 | p.m325.gB4 | p.m342.gC7 | p.m358.gG3 | p.m375.gA6 |
| p.m292.gC7 | p.m309.gF2 | p.m325.gB5 | p.m342.gD7 | p.m359.gF3 | p.m375.gB6 |
| p.m292.gD6 | p.m309.gF3 | p.m326.gA5 | p.m342.gD8 | p.m359.gG3 | p.m376.gB6 |
| p.m293.gC7 | p.m309.gG2 | p.m326.gB5 | p.m343.gD1 | p.m359.gG4 | p.m376.gA7 |
| p.m293.gC8 | p.m310.gF3 | p.m326.gB6 | p.m343.gE1 | p.m360.gF4 | p.m376.gB7 |
| p.m293.gD7 | p.m310.gF4 | p.m327.gA6 | p.m343.gE2 | p.m360.gG4 | p.m377.gB7 |
| p.m294.gD1 | p.m310.gG3 | p.m327.gB6 | p.m344.gD2 | p.m360.gG5 | p.m377.gA8 |
| p.m294.gD2 | p.m311.gF4 | p.m327.gB7 | p.m344.gE2 | p.m361.gF5 | p.m377.gB8 |
| p.m294.gE1 | p.m311.gF5 | p.m328.gA7 | p.m344.gE3 | p.m361.gG5 | p.m378.gC1 |
| p.m295.gD2 | p.m311.gG4 | p.m328.gB7 | p.m345.gD3 | p.m361.gG6 | p.m378.gB2 |
| p.m295.gD3 | p.m312.gF5 | p.m328.gB8 | p.m345.gE3 | p.m362.gF6 | p.m378.gC2 |
| p.m295.gE2 | p.m312.gF6 | p.m329.gB1 | p.m345.gE4 | p.m362.gG6 | p.m379.gC2 |
| p.m296.gD3 | p.m312.gG5 | p.m329.gC1 | p.m346.gD4 | p.m362.gG7 | p.m379.gB3 |
| p.m296.gD4 | p.m313.gF6 | p.m329.gC2 | p.m346.gE4 | p.m363.gF7 | p.m379.gC3 |
| p.m296.gE3 | p.m313.gF7 | p.m330.gB2 | p.m346.gE5 | p.m363.gG7 | p.m380.gC3 |
| p.m297.gD4 | p.m313.gG6 | p.m330.gC2 | p.m347.gD5 | p.m363.gG8 | p.m380.gB4 |
| p.m297.gD5 | p.m314.gF7 | p.m330.gC3 | p.m347.gE5 | p.m364.gG1 | p.m380.gC4 |

***G*** $_{pm}$ ***(for all p except SSN1 and SSN2) (continued)***

| | | | | | |
|---|---|---|---|---|---|
| p.m381.gC4 | p.m397.gE7 | p.m414.gG3 | p.m431.gB5 | p.m447.gE8 | p.m464.gG4 |
| p.m381.gB5 | p.m398.gE7 | p.m414.gH3 | p.m431.gB6 | p.m448.gE1 | p.m464.gH4 |
| p.m381.gC5 | p.m398.gD8 | p.m415.gH3 | p.m431.gC6 | p.m448.gE2 | p.m465.gG4 |
| p.m382.gC5 | p.m398.gE8 | p.m415.gG4 | p.m432.gB6 | p.m448.gF2 | p.m465.gG5 |
| p.m382.gB6 | p.m399.gF1 | p.m415.gH4 | p.m432.gB7 | p.m449.gE2 | p.m465.gH5 |
| p.m382.gC6 | p.m399.gE2 | p.m416.gH4 | p.m432.gC7 | p.m449.gE3 | p.m466.gG5 |
| p.m383.gC6 | p.m399.gF2 | p.m416.gG5 | p.m433.gB7 | p.m449.gF3 | p.m466.gG6 |
| p.m383.gB7 | p.m400.gF2 | p.m416.gH5 | p.m433.gB8 | p.m450.gE3 | p.m466.gH6 |
| p.m383.gC7 | p.m400.gE3 | p.m417.gH5 | p.m433.gC8 | p.m450.gE4 | p.m467.gG6 |
| p.m384.gC7 | p.m400.gF3 | p.m417.gG6 | p.m434.gC1 | p.m450.gF4 | p.m467.gG7 |
| p.m384.gB8 | p.m401.gF3 | p.m417.gH6 | p.m434.gC2 | p.m451.gE4 | p.m467.gH7 |
| p.m384.gC8 | p.m401.gE4 | p.m418.gH6 | p.m434.gD2 | p.m451.gE5 | p.m468.gG7 |
| p.m385.gD1 | p.m401.gF4 | p.m418.gG7 | p.m435.gC2 | p.m451.gF5 | p.m468.gG8 |
| p.m385.gC2 | p.m402.gF4 | p.m418.gH7 | p.m435.gC3 | p.m452.gE5 | p.m468.gH8 |
| p.m385.gD2 | p.m402.gE5 | p.m419.gH7 | p.m435.gD3 | p.m452.gE6 | |
| p.m386.gD2 | p.m402.gF5 | p.m419.gG8 | p.m436.gC3 | p.m452.gF6 | |
| p.m386.gC3 | p.m403.gF5 | p.m419.gH8 | p.m436.gC4 | p.m453.gE6 | |
| p.m386.gD3 | p.m403.gE6 | p.m420.gA1 | p.m436.gD4 | p.m453.gE7 | |
| p.m387.gD3 | p.m403.gF6 | p.m420.gA2 | p.m437.gC4 | p.m453.gF7 | |
| p.m387.gC4 | p.m404.gF6 | p.m420.gB2 | p.m437.gC5 | p.m454.gE7 | |
| p.m387.gD4 | p.m404.gE7 | p.m421.gA2 | p.m437.gD5 | p.m454.gE8 | |
| p.m388.gD4 | p.m404.gF7 | p.m421.gA3 | p.m438.gC5 | p.m454.gF8 | |
| p.m388.gC5 | p.m405.gF7 | p.m421.gB3 | p.m438.gC6 | p.m455.gF1 | |
| p.m388.gD5 | p.m405.gE8 | p.m422.gA3 | p.m438.gD6 | p.m455.gF2 | |
| p.m389.gD5 | p.m405.gF8 | p.m422.gA4 | p.m439.gC6 | p.m455.gG2 | |
| p.m389.gC6 | p.m406.gG1 | p.m422.gB4 | p.m439.gC7 | p.m456.gF2 | |
| p.m389.gD6 | p.m406.gF2 | p.m423.gA4 | p.m439.gD7 | p.m456.gF3 | |
| p.m390.gD6 | p.m406.gG2 | p.m423.gA5 | p.m440.gC7 | p.m456.gG3 | |
| p.m390.gC7 | p.m407.gG2 | p.m423.gB5 | p.m440.gC8 | p.m457.gF3 | |
| p.m390.gD7 | p.m407.gF3 | p.m424.gA5 | p.m440.gD8 | p.m457.gF4 | |
| p.m391.gD7 | p.m407.gG3 | p.m424.gA6 | p.m441.gD1 | p.m457.gG4 | |
| p.m391.gC8 | p.m408.gG3 | p.m424.gB6 | p.m441.gD2 | p.m458.gF4 | |
| p.m391.gD8 | p.m408.gF4 | p.m425.gA6 | p.m441.gE2 | p.m458.gF5 | |
| p.m392.gE1 | p.m408.gG4 | p.m425.gA7 | p.m442.gD2 | p.m458.gG5 | |
| p.m392.gD2 | p.m409.gG4 | p.m425.gB7 | p.m442.gD3 | p.m459.gF5 | |
| p.m392.gE2 | p.m409.gF5 | p.m426.gA7 | p.m442.gE3 | p.m459.gF6 | |
| p.m393.gE2 | p.m409.gG5 | p.m426.gA8 | p.m443.gD3 | p.m459.gG6 | |
| p.m393.gD3 | p.m410.gG5 | p.m426.gB8 | p.m443.gD4 | p.m460.gF6 | |
| p.m393.gE3 | p.m410.gF6 | p.m427.gB1 | p.m443.gE4 | p.m460.gF7 | |
| p.m394.gE3 | p.m410.gG6 | p.m427.gB2 | p.m444.gD4 | p.m460.gG7 | |
| p.m394.gD4 | p.m411.gG6 | p.m427.gC2 | p.m444.gD5 | p.m461.gF7 | |
| p.m394.gE4 | p.m411.gF7 | p.m428.gB2 | p.m444.gE5 | p.m461.gF8 | |
| p.m395.gE4 | p.m411.gG7 | p.m428.gB3 | p.m445.gD5 | p.m461.gG8 | |
| p.m395.gD5 | p.m412.gG7 | p.m428.gC3 | p.m445.gD6 | p.m462.gG1 | |
| p.m395.gE5 | p.m412.gF8 | p.m429.gB3 | p.m445.gE6 | p.m462.gG2 | |
| p.m396.gE5 | p.m412.gG8 | p.m429.gB4 | p.m446.gD6 | p.m462.gH2 | |
| p.m396.gD6 | p.m413.gH1 | p.m429.gC4 | p.m446.gD7 | p.m463.gG2 | |
| p.m396.gE6 | p.m413.gG2 | p.m430.gB4 | p.m446.gE7 | p.m463.gG3 | |
| p.m397.gE6 | p.m413.gH2 | p.m430.gB5 | p.m447.gD7 | p.m463.gH3 | |
| p.m397.gD7 | p.m414.gH2 | p.m430.gC5 | p.m447.gD8 | p.m464.gG3 | |

***$G_{pm}$ (for SSN1)***

| | | | | | |
|---|---|---|---|---|---|
| SSN1.m1.gC1 | SSN1.m30.gG3 | SSN1.m49.gE1 | SSN1.m62.gH5 | SSN1.m76.gF1 | SSN1.m89.gG3 |
| SSN1.m2.gC2 | SSN1.m31.gG3 | SSN1.m49.gF1 | SSN1.m63.gC1 | SSN1.m76.gG1 | SSN1.m89.gH3 |
| SSN1.m3.gD1 | SSN1.m31.gH3 | SSN1.m50.gD2 | SSN1.m63.gC2 | SSN1.m76.gG2 | SSN1.m90.gH4 |
| SSN1.m4.gD2 | SSN1.m32.gG4 | SSN1.m50.gE2 | SSN1.m63.gD1 | SSN1.m77.gF2 | SSN1.m90.gG5 |
| SSN1.m5.gE1 | SSN1.m32.gH4 | SSN1.m50.gF2 | SSN1.m64.gD1 | SSN1.m77.gG2 | SSN1.m90.gH5 |
| SSN1.m6.gE2 | SSN1.m33.gG5 | SSN1.m51.gE1 | SSN1.m64.gD2 | SSN1.m77.gG3 | SSN1.m91.gH3 |
| SSN1.m7.gF1 | SSN1.m33.gH5 | SSN1.m51.gF1 | SSN1.m64.gE1 | SSN1.m78.gG1 | SSN1.m91.gG4 |
| SSN1.m8.gF2 | SSN1.m34.gC1 | SSN1.m51.gG1 | SSN1.m65.gE1 | SSN1.m78.gH1 | SSN1.m91.gH4 |
| SSN1.m9.gG1 | SSN1.m34.gC2 | SSN1.m52.gE2 | SSN1.m65.gE2 | SSN1.m78.gH2 | SSN1.m92.gC1 |
| SSN1.m10.gG2 | SSN1.m35.gD1 | SSN1.m52.gF2 | SSN1.m65.gF1 | SSN1.m79.gG2 | SSN1.m92.gC2 |
| SSN1.m11.gH1 | SSN1.m35.gD2 | SSN1.m52.gG2 | SSN1.m66.gF1 | SSN1.m79.gH2 | SSN1.m92.gD2 |
| SSN1.m12.gH2 | SSN1.m36.gE1 | SSN1.m53.gF1 | SSN1.m66.gF2 | SSN1.m79.gH3 | SSN1.m93.gD1 |
| SSN1.m13.gF3 | SSN1.m36.gE2 | SSN1.m53.gG1 | SSN1.m66.gG1 | SSN1.m80.gF3 | SSN1.m93.gD2 |
| SSN1.m14.gG3 | SSN1.m37.gF1 | SSN1.m53.gH1 | SSN1.m67.gF2 | SSN1.m80.gG3 | SSN1.m93.gE2 |
| SSN1.m15.gG4 | SSN1.m37.gF2 | SSN1.m54.gF2 | SSN1.m67.gF3 | SSN1.m80.gG4 | SSN1.m94.gE1 |
| SSN1.m16.gG5 | SSN1.m38.gF2 | SSN1.m54.gG2 | SSN1.m67.gG2 | SSN1.m81.gG3 | SSN1.m94.gE2 |
| SSN1.m17.gH5 | SSN1.m38.gF3 | SSN1.m54.gH2 | SSN1.m68.gG1 | SSN1.m81.gH3 | SSN1.m94.gF2 |
| SSN1.m18.gH4 | SSN1.m39.gG1 | SSN1.m55.gF3 | SSN1.m68.gG2 | SSN1.m81.gH4 | SSN1.m95.gF1 |
| SSN1.m19.gH3 | SSN1.m39.gG2 | SSN1.m55.gG3 | SSN1.m68.gH1 | SSN1.m82.gG4 | SSN1.m95.gF2 |
| SSN1.m20.gC1 | SSN1.m40.gG2 | SSN1.m55.gH3 | SSN1.m69.gG2 | SSN1.m82.gH4 | SSN1.m95.gG2 |
| SSN1.m20.gD1 | SSN1.m40.gG3 | SSN1.m56.gF1 | SSN1.m69.gG3 | SSN1.m82.gH5 | SSN1.m96.gF2 |
| SSN1.m21.gC2 | SSN1.m41.gH1 | SSN1.m56.gF2 | SSN1.m69.gH2 | SSN1.m83.gD1 | SSN1.m96.gF3 |
| SSN1.m21.gD2 | SSN1.m41.gH2 | SSN1.m56.gF3 | SSN1.m70.gG3 | SSN1.m83.gC2 | SSN1.m96.gG3 |
| SSN1.m22.gD1 | SSN1.m42.gH2 | SSN1.m57.gG1 | SSN1.m70.gG4 | SSN1.m83.gD2 | SSN1.m97.gG1 |
| SSN1.m22.gE1 | SSN1.m42.gH3 | SSN1.m57.gG2 | SSN1.m70.gH3 | SSN1.m84.gE1 | SSN1.m97.gG2 |
| SSN1.m23.gD2 | SSN1.m43.gG3 | SSN1.m57.gG3 | SSN1.m71.gG4 | SSN1.m84.gD2 | SSN1.m97.gH2 |
| SSN1.m23.gE2 | SSN1.m43.gG4 | SSN1.m58.gG2 | SSN1.m71.gG5 | SSN1.m84.gE2 | SSN1.m98.gG2 |
| SSN1.m24.gE1 | SSN1.m44.gG4 | SSN1.m58.gG3 | SSN1.m71.gH4 | SSN1.m85.gF1 | SSN1.m98.gG3 |
| SSN1.m24.gF1 | SSN1.m44.gG5 | SSN1.m58.gG4 | SSN1.m72.gC1 | SSN1.m85.gE2 | SSN1.m98.gH3 |
| SSN1.m25.gE2 | SSN1.m45.gH4 | SSN1.m59.gH1 | SSN1.m72.gD1 | SSN1.m85.gF2 | SSN1.m99.gG3 |
| SSN1.m25.gF2 | SSN1.m45.gH5 | SSN1.m59.gH2 | SSN1.m72.gD2 | SSN1.m86.gG1 | SSN1.m99.gG4 |
| SSN1.m26.gF1 | SSN1.m46.gH3 | SSN1.m59.gH3 | SSN1.m73.gD1 | SSN1.m86.gF2 | SSN1.m99.gH4 |
| SSN1.m26.gG1 | SSN1.m46.gH4 | SSN1.m60.gH2 | SSN1.m73.gE1 | SSN1.m86.gG2 | SSN1.m100.gG4 |
| SSN1.m27.gF2 | SSN1.m47.gC1 | SSN1.m60.gH3 | SSN1.m73.gE2 | SSN1.m87.gG2 | SSN1.m100.gG5 |
| SSN1.m27.gG2 | SSN1.m47.gD1 | SSN1.m60.gH4 | SSN1.m74.gE1 | SSN1.m87.gF3 | SSN1.m100.gH5 |
| SSN1.m28.gG1 | SSN1.m47.gE1 | SSN1.m61.gG3 | SSN1.m74.gF1 | SSN1.m87.gG3 | |
| SSN1.m28.gH1 | SSN1.m48.gC2 | SSN1.m61.gG4 | SSN1.m74.gF2 | SSN1.m88.gH1 | |
| SSN1.m29.gG2 | SSN1.m48.gD2 | SSN1.m61.gG5 | SSN1.m75.gE2 | SSN1.m88.gG2 | |
| SSN1.m29.gH2 | SSN1.m48.gE2 | SSN1.m62.gH3 | SSN1.m75.gF2 | SSN1.m88.gH2 | |
| SSN1.m30.gF3 | SSN1.m49.gD1 | SSN1.m62.gH4 | SSN1.m75.gF3 | SSN1.m89.gH2 | |

***$G_{pm}$ (for SSN2)***

| | | | | | |
|---|---|---|---|---|---|
| SSN2.m1.gA4 | SSN2.m38.gC4 | SSN2.m60.gA5 | SSN2.m81.gF7 | SSN2.m97.gC6 | SSN2.m111.gC5 |
| SSN2.m2.gA5 | SSN2.m39.gB5 | SSN2.m60.gA6 | SSN2.m82.gF7 | SSN2.m97.gD6 | SSN2.m111.gC6 |
| SSN2.m3.gA6 | SSN2.m39.gC5 | SSN2.m61.gA6 | SSN2.m82.gF8 | SSN2.m97.gE6 | SSN2.m112.gC5 |
| SSN2.m4.gA7 | SSN2.m40.gB6 | SSN2.m61.gA7 | SSN2.m83.gD3 | SSN2.m98.gC7 | SSN2.m112.gC6 |
| SSN2.m5.gA8 | SSN2.m40.gC6 | SSN2.m62.gA7 | SSN2.m83.gD4 | SSN2.m98.gD7 | SSN2.m112.gC7 |
| SSN2.m6.gB4 | SSN2.m41.gB7 | SSN2.m62.gA8 | SSN2.m84.gE3 | SSN2.m98.gE7 | SSN2.m113.gC6 |
| SSN2.m7.gB5 | SSN2.m41.gC7 | SSN2.m63.gB4 | SSN2.m84.gE4 | SSN2.m99.gC8 | SSN2.m113.gC7 |
| SSN2.m8.gB6 | SSN2.m42.gB8 | SSN2.m63.gB5 | SSN2.m85.gA4 | SSN2.m99.gD8 | SSN2.m113.gC8 |
| SSN2.m9.gB7 | SSN2.m42.gC8 | SSN2.m64.gB5 | SSN2.m85.gB4 | SSN2.m99.gE8 | SSN2.m114.gD4 |
| SSN2.m10.gB8 | SSN2.m43.gC4 | SSN2.m64.gB6 | SSN2.m85.gC4 | SSN2.m100.gD4 | SSN2.m114.gD5 |
| SSN2.m11.gC4 | SSN2.m43.gD4 | SSN2.m65.gB6 | SSN2.m86.gA5 | SSN2.m100.gE4 | SSN2.m114.gD6 |
| SSN2.m12.gC5 | SSN2.m44.gC5 | SSN2.m65.gB7 | SSN2.m86.gB5 | SSN2.m100.gF4 | SSN2.m115.gD5 |
| SSN2.m13.gC6 | SSN2.m44.gD5 | SSN2.m66.gB7 | SSN2.m86.gC5 | SSN2.m101.gD5 | SSN2.m115.gD6 |
| SSN2.m14.gC7 | SSN2.m45.gC6 | SSN2.m66.gB8 | SSN2.m87.gA6 | SSN2.m101.gE5 | SSN2.m115.gD7 |
| SSN2.m15.gC8 | SSN2.m45.gD6 | SSN2.m67.gC4 | SSN2.m87.gB6 | SSN2.m101.gF5 | SSN2.m116.gD6 |
| SSN2.m16.gD4 | SSN2.m46.gC7 | SSN2.m67.gC5 | SSN2.m87.gC6 | SSN2.m102.gD6 | SSN2.m116.gD7 |
| SSN2.m17.gD5 | SSN2.m46.gD7 | SSN2.m68.gC5 | SSN2.m88.gA7 | SSN2.m102.gE6 | SSN2.m116.gD8 |
| SSN2.m18.gD6 | SSN2.m47.gC8 | SSN2.m68.gC6 | SSN2.m88.gB7 | SSN2.m102.gF6 | SSN2.m117.gE4 |
| SSN2.m19.gD7 | SSN2.m47.gD8 | SSN2.m69.gC6 | SSN2.m88.gC7 | SSN2.m103.gD7 | SSN2.m117.gE5 |
| SSN2.m20.gD8 | SSN2.m48.gD4 | SSN2.m69.gC7 | SSN2.m89.gA8 | SSN2.m103.gE7 | SSN2.m117.gE6 |
| SSN2.m21.gE4 | SSN2.m48.gE4 | SSN2.m70.gC7 | SSN2.m89.gB8 | SSN2.m103.gF7 | SSN2.m118.gE5 |
| SSN2.m22.gE5 | SSN2.m49.gD5 | SSN2.m70.gC8 | SSN2.m89.gC8 | SSN2.m104.gD8 | SSN2.m118.gE6 |
| SSN2.m23.gE6 | SSN2.m49.gE5 | SSN2.m71.gD4 | SSN2.m90.gB4 | SSN2.m104.gE8 | SSN2.m118.gE7 |
| SSN2.m24.gE7 | SSN2.m50.gD6 | SSN2.m71.gD5 | SSN2.m90.gC4 | SSN2.m104.gF8 | SSN2.m119.gE6 |
| SSN2.m25.gE8 | SSN2.m50.gE6 | SSN2.m72.gD5 | SSN2.m90.gD4 | SSN2.m105.gA4 | SSN2.m119.gE7 |
| SSN2.m26.gF4 | SSN2.m51.gD7 | SSN2.m72.gD6 | SSN2.m91.gB5 | SSN2.m105.gA5 | SSN2.m119.gE8 |
| SSN2.m27.gF5 | SSN2.m51.gE7 | SSN2.m73.gD6 | SSN2.m91.gC5 | SSN2.m105.gA6 | SSN2.m120.gF4 |
| SSN2.m28.gF6 | SSN2.m52.gD8 | SSN2.m73.gD7 | SSN2.m91.gD5 | SSN2.m106.gA5 | SSN2.m120.gF5 |
| SSN2.m29.gF7 | SSN2.m52.gE8 | SSN2.m74.gD7 | SSN2.m92.gB6 | SSN2.m106.gA6 | SSN2.m120.gF6 |
| SSN2.m30.gF8 | SSN2.m53.gE4 | SSN2.m74.gD8 | SSN2.m92.gC6 | SSN2.m106.gA7 | SSN2.m121.gF5 |
| SSN2.m31.gD3 | SSN2.m53.gF4 | SSN2.m75.gE4 | SSN2.m92.gD6 | SSN2.m107.gA6 | SSN2.m121.gF6 |
| SSN2.m32.gE3 | SSN2.m54.gE5 | SSN2.m75.gE5 | SSN2.m93.gB7 | SSN2.m107.gA7 | SSN2.m121.gF7 |
| SSN2.m33.gA4 | SSN2.m54.gF5 | SSN2.m76.gE5 | SSN2.m93.gC7 | SSN2.m107.gA8 | SSN2.m122.gF6 |
| SSN2.m33.gB4 | SSN2.m55.gE6 | SSN2.m76.gE6 | SSN2.m93.gD7 | SSN2.m108.gB4 | SSN2.m122.gF7 |
| SSN2.m34.gA5 | SSN2.m55.gF6 | SSN2.m77.gE6 | SSN2.m94.gB8 | SSN2.m108.gB5 | SSN2.m122.gF8 |
| SSN2.m34.gB5 | SSN2.m56.gE7 | SSN2.m77.gE7 | SSN2.m94.gC8 | SSN2.m108.gB6 | SSN2.m123.gD3 |
| SSN2.m35.gA6 | SSN2.m56.gF7 | SSN2.m78.gE7 | SSN2.m94.gD8 | SSN2.m109.gB5 | SSN2.m123.gD4 |
| SSN2.m35.gB6 | SSN2.m57.gE8 | SSN2.m78.gE8 | SSN2.m95.gC4 | SSN2.m109.gB6 | SSN2.m123.gD5 |
| SSN2.m36.gA7 | SSN2.m57.gF8 | SSN2.m79.gF4 | SSN2.m95.gD4 | SSN2.m109.gB7 | SSN2.m124.gE3 |
| SSN2.m36.gB7 | SSN2.m58.gD3 | SSN2.m79.gF5 | SSN2.m95.gE4 | SSN2.m110.gB6 | SSN2.m124.gE4 |
| SSN2.m37.gA8 | SSN2.m58.gE3 | SSN2.m80.gF5 | SSN2.m96.gC5 | SSN2.m110.gB7 | SSN2.m124.gE5 |
| SSN2.m37.gB8 | SSN2.m59.gA4 | SSN2.m80.gF6 | SSN2.m96.gD5 | SSN2.m110.gB8 | SSN2.m125.gA4 |
| SSN2.m38.gB4 | SSN2.m59.gA5 | SSN2.m81.gF6 | SSN2.m96.gE5 | SSN2.m111.gC4 | SSN2.m125.gA5 |

***$G_{pm}$ (for SSN2) (continued)***

| | | | | | |
|---|---|---|---|---|---|
| SSN2.m125.gB4 | SSN2.m140.gD7 | SSN2.m154.gD4 | SSN2.m168.gB6 | SSN2.m183.gF4 | SSN2.m197.gC5 |
| SSN2.m126.gA5 | SSN2.m140.gD8 | SSN2.m154.gD5 | SSN2.m169.gB6 | SSN2.m183.gE5 | SSN2.m197.gD5 |
| SSN2.m126.gA6 | SSN2.m140.gE7 | SSN2.m155.gC5 | SSN2.m169.gA7 | SSN2.m183.gF5 | SSN2.m198.gC5 |
| SSN2.m126.gB5 | SSN2.m141.gE4 | SSN2.m155.gD5 | SSN2.m169.gB7 | SSN2.m184.gF5 | SSN2.m198.gC6 |
| SSN2.m127.gA6 | SSN2.m141.gE5 | SSN2.m155.gD6 | SSN2.m170.gB7 | SSN2.m184.gE6 | SSN2.m198.gD6 |
| SSN2.m127.gA7 | SSN2.m141.gF4 | SSN2.m156.gC6 | SSN2.m170.gA8 | SSN2.m184.gF6 | SSN2.m199.gC6 |
| SSN2.m127.gB6 | SSN2.m142.gE5 | SSN2.m156.gD6 | SSN2.m170.gB8 | SSN2.m185.gF6 | SSN2.m199.gC7 |
| SSN2.m128.gA7 | SSN2.m142.gE6 | SSN2.m156.gD7 | SSN2.m171.gC4 | SSN2.m185.gE7 | SSN2.m199.gD7 |
| SSN2.m128.gA8 | SSN2.m142.gF5 | SSN2.m157.gC7 | SSN2.m171.gB5 | SSN2.m185.gF7 | SSN2.m200.gC7 |
| SSN2.m128.gB7 | SSN2.m143.gE6 | SSN2.m157.gD7 | SSN2.m171.gC5 | SSN2.m186.gF7 | SSN2.m200.gC8 |
| SSN2.m129.gB4 | SSN2.m143.gE7 | SSN2.m157.gD8 | SSN2.m172.gC5 | SSN2.m186.gE8 | SSN2.m200.gD8 |
| SSN2.m129.gB5 | SSN2.m143.gF6 | SSN2.m158.gD4 | SSN2.m172.gB6 | SSN2.m186.gF8 | SSN2.m201.gD4 |
| SSN2.m129.gC4 | SSN2.m144.gE7 | SSN2.m158.gE4 | SSN2.m172.gC6 | SSN2.m187.gD3 | SSN2.m201.gD5 |
| SSN2.m130.gB5 | SSN2.m144.gE8 | SSN2.m158.gE5 | SSN2.m173.gC6 | SSN2.m187.gC4 | SSN2.m201.gE5 |
| SSN2.m130.gB6 | SSN2.m144.gF7 | SSN2.m159.gD5 | SSN2.m173.gB7 | SSN2.m187.gD4 | SSN2.m202.gD5 |
| SSN2.m130.gC5 | SSN2.m145.gD3 | SSN2.m159.gE5 | SSN2.m173.gC7 | SSN2.m188.gE3 | SSN2.m202.gD6 |
| SSN2.m131.gB6 | SSN2.m145.gD4 | SSN2.m159.gE6 | SSN2.m174.gC7 | SSN2.m188.gD4 | SSN2.m202.gE6 |
| SSN2.m131.gB7 | SSN2.m145.gE3 | SSN2.m160.gD6 | SSN2.m174.gB8 | SSN2.m188.gE4 | SSN2.m203.gD6 |
| SSN2.m131.gC6 | SSN2.m146.gA4 | SSN2.m160.gE6 | SSN2.m174.gC8 | SSN2.m189.gA4 | SSN2.m203.gD7 |
| SSN2.m132.gB7 | SSN2.m146.gB4 | SSN2.m160.gE7 | SSN2.m175.gD4 | SSN2.m189.gA5 | SSN2.m203.gE7 |
| SSN2.m132.gB8 | SSN2.m146.gB5 | SSN2.m161.gD7 | SSN2.m175.gC5 | SSN2.m189.gB5 | SSN2.m204.gD7 |
| SSN2.m132.gC7 | SSN2.m147.gA5 | SSN2.m161.gE7 | SSN2.m175.gD5 | SSN2.m190.gA5 | SSN2.m204.gD8 |
| SSN2.m133.gC4 | SSN2.m147.gB5 | SSN2.m161.gE8 | SSN2.m176.gD5 | SSN2.m190.gA6 | SSN2.m204.gE8 |
| SSN2.m133.gC5 | SSN2.m147.gB6 | SSN2.m162.gE4 | SSN2.m176.gC6 | SSN2.m190.gB6 | SSN2.m205.gE4 |
| SSN2.m133.gD4 | SSN2.m148.gA6 | SSN2.m162.gF4 | SSN2.m176.gD6 | SSN2.m191.gA6 | SSN2.m205.gE5 |
| SSN2.m134.gC5 | SSN2.m148.gB6 | SSN2.m162.gF5 | SSN2.m177.gD6 | SSN2.m191.gA7 | SSN2.m205.gF5 |
| SSN2.m134.gC6 | SSN2.m148.gB7 | SSN2.m163.gE5 | SSN2.m177.gC7 | SSN2.m191.gB7 | SSN2.m206.gE5 |
| SSN2.m134.gD5 | SSN2.m149.gA7 | SSN2.m163.gF5 | SSN2.m177.gD7 | SSN2.m192.gA7 | SSN2.m206.gE6 |
| SSN2.m135.gC6 | SSN2.m149.gB7 | SSN2.m163.gF6 | SSN2.m178.gD7 | SSN2.m192.gA8 | SSN2.m206.gF6 |
| SSN2.m135.gC7 | SSN2.m149.gB8 | SSN2.m164.gE6 | SSN2.m178.gC8 | SSN2.m192.gB8 | SSN2.m207.gE6 |
| SSN2.m135.gD6 | SSN2.m150.gB4 | SSN2.m164.gF6 | SSN2.m178.gD8 | SSN2.m193.gB4 | SSN2.m207.gE7 |
| SSN2.m136.gC7 | SSN2.m150.gC4 | SSN2.m164.gF7 | SSN2.m179.gE4 | SSN2.m193.gB5 | SSN2.m207.gF7 |
| SSN2.m136.gC8 | SSN2.m150.gC5 | SSN2.m165.gE7 | SSN2.m179.gD5 | SSN2.m193.gC5 | SSN2.m208.gE7 |
| SSN2.m136.gD7 | SSN2.m151.gB5 | SSN2.m165.gF7 | SSN2.m179.gE5 | SSN2.m194.gB5 | SSN2.m208.gE8 |
| SSN2.m137.gD4 | SSN2.m151.gC5 | SSN2.m165.gF8 | SSN2.m180.gE5 | SSN2.m194.gB6 | SSN2.m208.gF8 |
| SSN2.m137.gD5 | SSN2.m151.gC6 | SSN2.m166.gD3 | SSN2.m180.gD6 | SSN2.m194.gC6 | SSN2.m209.gD3 |
| SSN2.m137.gE4 | SSN2.m152.gB6 | SSN2.m166.gE3 | SSN2.m180.gE6 | SSN2.m195.gB6 | SSN2.m209.gD4 |
| SSN2.m138.gD5 | SSN2.m152.gC6 | SSN2.m166.gE4 | SSN2.m181.gE6 | SSN2.m195.gB7 | SSN2.m209.gE4 |
| SSN2.m138.gD6 | SSN2.m152.gC7 | SSN2.m167.gB4 | SSN2.m181.gD7 | SSN2.m195.gC7 | SSN2.m210.gE3 |
| SSN2.m138.gE5 | SSN2.m153.gB7 | SSN2.m167.gA5 | SSN2.m181.gE7 | SSN2.m196.gB7 | SSN2.m210.gE4 |
| SSN2.m139.gD6 | SSN2.m153.gC7 | SSN2.m167.gB5 | SSN2.m182.gE7 | SSN2.m196.gB8 | SSN2.m210.gF4 |
| SSN2.m139.gD7 | SSN2.m153.gC8 | SSN2.m168.gB5 | SSN2.m182.gD8 | SSN2.m196.gC8 | |
| SSN2.m139.gE6 | SSN2.m154.gC4 | SSN2.m168.gA6 | SSN2.m182.gE8 | SSN2.m197.gC4 | |

## Data

***dist*** $_q$

| | |
|---|---|
| gA1 | 25.0 |
| gA2 | 20.0 |
| gA3 | 15.0 |
| gA4 | 10.0 |
| gA5 | 10.0 |
| gA6 | 10.0 |
| gA7 | 10.0 |
| gA8 | 10.0 |
| gB1 | 20.0 |
| gB2 | 20.0 |
| gB3 | 15.0 |
| gB4 | 10.0 |
| gB5 | 5.0 |
| gB6 | 5.0 |
| gB7 | 5.0 |
| gB8 | 10.0 |
| gC1 | 15.0 |
| gC2 | 15.0 |
| gC3 | 15.0 |
| gC4 | 10.0 |
| gC5 | 5.0 |
| gC6 | 0.0 |
| gC7 | 5.0 |
| gC8 | 10.0 |
| gD1 | 10.0 |
| gD2 | 10.0 |
| gD3 | 10.0 |
| gD4 | 10.0 |
| gD5 | 5.0 |
| gD6 | 5.0 |
| gD7 | 5.0 |
| gD8 | 10.0 |
| gE1 | 10.0 |
| gE2 | 5.0 |
| gE3 | 5.0 |
| gE4 | 5.0 |
| gE5 | 10.0 |
| gE6 | 10.0 |
| gE7 | 10.0 |
| gE8 | 10.0 |
| gF1 | 10.0 |
| gF2 | 5.0 |
| gF3 | 0.0 |
| gF4 | 5.0 |
| gF5 | 10.0 |
| gF6 | 15.0 |
| gF7 | 15.0 |
| gF8 | 15.0 |
| gG1 | 10.0 |
| gG2 | 5.0 |
| gG3 | 5.0 |
| gG4 | 5.0 |
| gG5 | 10.0 |
| gG6 | 15.0 |
| gG7 | 20.0 |
| gG8 | 20.0 |
| gH1 | 10.0 |
| gH2 | 10.0 |
| gH3 | 10.0 |
| gH4 | 10.0 |
| gH5 | 10.0 |
| gH6 | 15.0 |
| gH7 | 20.0 |
| gH8 | 25.0 |

***teth_range*** $_{p''}$

| | |
|---|---|
| Helo1 | 10.0 |
| Helo2 | 10.0 |

***trans*** $_p$

| | |
|---|---|
| Surf1 | 0.5 |
| Surf2 | 0.5 |
| Surf3 | 0.5 |
| Helo1 | 0.1 |
| Helo2 | 0.1 |
| P31 | 0.1 |
| SSN1 | 0.7 |
| SSN2 | 0.7 |

***r*** $_{psq}$

| | Surf1.active | Surf2.active | Surf3.active | Helo1.active | Helo2.active | P31.active | SSN1.passive | SSN1.active | SSN2.passive |
|---|---|---|---|---|---|---|---|---|---|
| gA1 | 0.41 | 0.41 | 0.41 | 0.49 | 0.49 | 0.53 | 0.44 | 0.61 | 0.44 |
| gA2 | 0.46 | 0.46 | 0.46 | 0.55 | 0.55 | 0.6 | 0.5 | 0.69 | 0.5 |
| gA3 | 0.39 | 0.39 | 0.39 | 0.47 | 0.47 | 0.51 | 0.42 | 0.59 | 0.42 |
| gA4 | 0.53 | 0.53 | 0.53 | 0.64 | 0.64 | 0.69 | 0.57 | 0.8 | 0.57 |
| gA5 | 0.73 | 0.73 | 0.73 | 0.87 | 0.87 | 0.95 | 0.78 | 1.09 | 0.78 |
| gA6 | 0.95 | 0.95 | 0.95 | 1.14 | 1.14 | 1.24 | 1.02 | 1.43 | 1.02 |
| gA7 | 0.83 | 0.83 | 0.83 | 0.99 | 0.99 | 1.07 | 0.89 | 1.24 | 0.89 |
| gA8 | 0.92 | 0.92 | 0.92 | 1.11 | 1.11 | 1.2 | 0.99 | 1.39 | 0.99 |
| gB1 | 0.43 | 0.43 | 0.43 | 0.52 | 0.52 | 0.56 | 0.47 | 0.65 | 0.47 |
| gB2 | 0.42 | 0.42 | 0.42 | 0.5 | 0.5 | 0.55 | 0.45 | 0.63 | 0.45 |
| gB3 | 0.53 | 0.53 | 0.53 | 0.64 | 0.64 | 0.69 | 0.57 | 0.8 | 0.57 |
| gB4 | 0.56 | 0.56 | 0.56 | 0.67 | 0.67 | 0.73 | 0.6 | 0.84 | 0.6 |
| gB5 | 0.7 | 0.7 | 0.7 | 0.84 | 0.84 | 0.91 | 0.75 | 1.05 | 0.75 |
| gB6 | 0.91 | 0.91 | 0.91 | 1.09 | 1.09 | 1.18 | 0.97 | 1.36 | 0.97 |
| gB7 | 0.95 | 0.95 | 0.95 | 1.14 | 1.14 | 1.24 | 1.02 | 1.43 | 1.02 |
| gB8 | 0.97 | 0.97 | 0.97 | 1.16 | 1.16 | 1.26 | 1.04 | 1.45 | 1.04 |
| gC1 | 0.48 | 0.48 | 0.48 | 0.57 | 0.57 | 0.62 | 0.51 | 0.71 | 0.51 |
| gC2 | 0.56 | 0.56 | 0.56 | 0.67 | 0.67 | 0.73 | 0.6 | 0.84 | 0.6 |
| gC3 | 0.49 | 0.49 | 0.49 | 0.59 | 0.59 | 0.64 | 0.53 | 0.74 | 0.53 |
| gC4 | 0.56 | 0.56 | 0.56 | 0.67 | 0.67 | 0.73 | 0.6 | 0.84 | 0.6 |
| gC5 | 0.7 | 0.7 | 0.7 | 0.84 | 0.84 | 0.91 | 0.75 | 1.05 | 0.75 |
| gC6 | 0.88 | 0.88 | 0.88 | 1.05 | 1.05 | 1.14 | 0.94 | 1.32 | 0.94 |
| gC7 | 1.03 | 1.03 | 1.03 | 1.24 | 1.24 | 1.34 | 1.11 | 1.55 | 1.11 |
| gC8 | 0.99 | 0.99 | 0.99 | 1.19 | 1.19 | 1.29 | 1.07 | 1.49 | 1.07 |
| gD1 | 0.59 | 0.59 | 0.59 | 0.71 | 0.71 | 0.76 | 0.63 | 0.88 | 0.63 |
| gD2 | 0.56 | 0.56 | 0.56 | 0.67 | 0.67 | 0.73 | 0.6 | 0.84 | 0.6 |
| gD3 | 0.63 | 0.63 | 0.63 | 0.76 | 0.76 | 0.82 | 0.68 | 0.95 | 0.68 |
| gD4 | 0.7 | 0.7 | 0.7 | 0.84 | 0.84 | 0.91 | 0.75 | 1.05 | 0.75 |
| gD5 | 0.84 | 0.84 | 0.84 | 1.01 | 1.01 | 1.09 | 0.9 | 1.26 | 0.9 |
| gD6 | 0.91 | 0.91 | 0.91 | 1.1 | 1.1 | 1.19 | 0.98 | 1.37 | 0.98 |
| gD7 | 0.97 | 0.97 | 0.96 | 1.16 | 1.16 | 1.26 | 1.03 | 1.45 | 1.03 |
| gD8 | 0.98 | 0.98 | 0.98 | 1.18 | 1.18 | 1.27 | 1.05 | 1.47 | 1.05 |
| gE1 | 0.95 | 0.95 | 0.95 | 1.14 | 1.14 | 1.24 | 1.02 | 1.43 | 1.02 |
| gE2 | 0.91 | 0.91 | 0.91 | 1.09 | 1.09 | 1.18 | 0.98 | 1.37 | 0.98 |
| gE3 | 1.01 | 1.01 | 1.01 | 1.22 | 1.22 | 1.32 | 1.08 | 1.52 | 1.08 |
| gE4 | 0.77 | 0.77 | 0.77 | 0.92 | 0.92 | 1 | 0.83 | 1.16 | 0.83 |
| gE5 | 0.63 | 0.63 | 0.63 | 0.76 | 0.76 | 0.82 | 0.68 | 0.95 | 0.68 |
| gE6 | 0.7 | 0.7 | 0.7 | 0.84 | 0.84 | 0.91 | 0.75 | 1.05 | 0.75 |
| gE7 | 0.7 | 0.7 | 0.7 | 0.84 | 0.84 | 0.91 | 0.75 | 1.05 | 0.75 |
| gE8 | 0.85 | 0.85 | 0.85 | 1.02 | 1.02 | 1.11 | 0.92 | 1.28 | 0.92 |
| gF1 | 0.87 | 0.87 | 0.87 | 1.04 | 1.04 | 1.13 | 0.93 | 1.3 | 0.93 |
| gF2 | 0.99 | 0.99 | 0.99 | 1.19 | 1.19 | 1.29 | 1.06 | 1.48 | 1.06 |
| gF3 | 1.03 | 1.03 | 1.03 | 1.24 | 1.24 | 1.34 | 1.1 | 1.55 | 1.1 |
| gF4 | 0.63 | 0.63 | 0.63 | 0.76 | 0.76 | 0.82 | 0.68 | 0.95 | 0.68 |
| gF5 | 0.49 | 0.49 | 0.49 | 0.59 | 0.59 | 0.64 | 0.53 | 0.74 | 0.53 |
| gF6 | 0.56 | 0.56 | 0.56 | 0.67 | 0.67 | 0.73 | 0.6 | 0.84 | 0.6 |
| gF7 | 0.56 | 0.56 | 0.56 | 0.67 | 0.67 | 0.73 | 0.6 | 0.84 | 0.6 |
| gF8 | 0.66 | 0.66 | 0.66 | 0.79 | 0.79 | 0.86 | 0.71 | 0.99 | 0.71 |
| gG1 | 0.99 | 0.99 | 0.99 | 1.19 | 1.19 | 1.29 | 1.07 | 1.49 | 1.07 |
| gG2 | 0.95 | 0.95 | 0.95 | 1.14 | 1.14 | 1.24 | 1.02 | 1.43 | 1.02 |
| gG3 | 0.88 | 0.88 | 0.88 | 1.05 | 1.05 | 1.14 | 0.94 | 1.31 | 0.94 |
| gG4 | 0.56 | 0.56 | 0.56 | 0.67 | 0.67 | 0.73 | 0.6 | 0.84 | 0.6 |
| gG5 | 0.42 | 0.42 | 0.42 | 0.5 | 0.5 | 0.55 | 0.45 | 0.63 | 0.45 |
| gG6 | 0.42 | 0.42 | 0.42 | 0.5 | 0.5 | 0.55 | 0.45 | 0.63 | 0.45 |
| gG7 | 0.42 | 0.42 | 0.42 | 0.5 | 0.5 | 0.55 | 0.45 | 0.63 | 0.45 |
| gG8 | 0.46 | 0.46 | 0.46 | 0.55 | 0.55 | 0.6 | 0.5 | 0.69 | 0.5 |
| gH1 | 0.97 | 0.97 | 0.97 | 1.16 | 1.16 | 1.26 | 1.04 | 1.45 | 1.04 |
| gH2 | 0.98 | 0.98 | 0.98 | 1.18 | 1.18 | 1.27 | 1.05 | 1.47 | 1.05 |
| gH3 | 0.84 | 0.84 | 0.84 | 1.01 | 1.01 | 1.09 | 0.9 | 1.26 | 0.9 |
| gH4 | 0.67 | 0.67 | 0.67 | 0.81 | 0.81 | 0.87 | 0.72 | 1.01 | 0.72 |
| gH5 | 0.41 | 0.41 | 0.41 | 0.49 | 0.49 | 0.53 | 0.44 | 0.61 | 0.44 |
| gH6 | 0.57 | 0.57 | 0.57 | 0.69 | 0.69 | 0.75 | 0.62 | 0.86 | 0.62 |
| gH7 | 0.49 | 0.49 | 0.49 | 0.59 | 0.59 | 0.64 | 0.53 | 0.74 | 0.53 |
| gH8 | 0.53 | 0.53 | 0.53 | 0.64 | 0.64 | 0.69 | 0.57 | 0.8 | 0.57 |

***range** $_{m'm''}$*

Determined from table below

e.g., for range(m1, m83), take the minimum of the distances (gA1, gC3), (gA1, gD3), which are 14.1, 18. Therefore, range(m1, m83) is 14.1.

| | gA1 | gA2 | gA3 | gA4 | gA5 | gA6 | gA7 | gA8 | gB1 | gB2 | gB3 | gB4 | gB5 | gB6 | gB7 | gB8 | gC1 | gC2 | gC3 | gC4 | gC5 | gC6 | gC7 | gC8 | gD1 | gD2 | gD3 | gD4 | gD5 | gD6 | gD7 | gD8 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| gA1 | 0 | 5 | 10 | 15 | 20 | 25 | 30 | 35 | 5 | 7.1 | 11.2 | 15.8 | 20.6 | 25.5 | 30.4 | 35.4 | 10 | 11.2 | 14.1 | 18 | 22.4 | 26.9 | 31.6 | 36.4 | 15 | 15.8 | 18 | 21.2 | 25 | 29.2 | 33.5 | 38.1 |
| gA2 | 5 | 0 | 5 | 10 | 15 | 20 | 25 | 30 | 7.1 | 5 | 7.1 | 11.2 | 15.8 | 20.6 | 25.5 | 30.4 | 11.2 | 10 | 11.2 | 14.1 | 18 | 22.4 | 26.9 | 31.6 | 15.8 | 15 | 15.8 | 18 | 21.2 | 25 | 29.2 | 33.5 |
| gA3 | 10 | 5 | 0 | 5 | 10 | 15 | 20 | 25 | 11.2 | 7.1 | 5 | 7.1 | 11.2 | 15.8 | 20.6 | 25.5 | 14.1 | 11.2 | 10 | 11.2 | 14.1 | 18 | 22.4 | 26.9 | 18 | 15.8 | 15 | 15.8 | 18 | 21.2 | 25 | 29.2 |
| gA4 | 15 | 10 | 5 | 0 | 5 | 10 | 15 | 20 | 15.8 | 11.2 | 7.1 | 5 | 7.1 | 11.2 | 15.8 | 20.6 | 18 | 14.1 | 11.2 | 10 | 11.2 | 14.1 | 18 | 22.4 | 21.2 | 18 | 15.8 | 15 | 15.8 | 18 | 21.2 | 25 |
| gA5 | 20 | 15 | 10 | 5 | 0 | 5 | 10 | 15 | 20.6 | 15.8 | 11.2 | 7.1 | 5 | 7.1 | 11.2 | 15.8 | 22.4 | 18 | 14.1 | 11.2 | 10 | 11.2 | 14.1 | 18 | 25 | 21.2 | 18 | 15.8 | 15 | 15.8 | 18 | 21.2 |
| gA6 | 25 | 20 | 15 | 10 | 5 | 0 | 5 | 10 | 25.5 | 20.6 | 15.8 | 11.2 | 7.1 | 5 | 7.1 | 11.2 | 26.9 | 22.4 | 18 | 14.1 | 11.2 | 10 | 11.2 | 14.1 | 29.2 | 25 | 21.2 | 18 | 15.8 | 15 | 15.8 | 18 |
| gA7 | 30 | 25 | 20 | 15 | 10 | 5 | 0 | 5 | 30.4 | 25.5 | 20.6 | 15.8 | 11.2 | 7.1 | 5 | 7.1 | 31.6 | 26.9 | 22.4 | 18 | 14.1 | 11.2 | 10 | 11.2 | 33.5 | 29.2 | 25 | 21.2 | 18 | 15.8 | 15 | 15.8 |
| gA8 | 35 | 30 | 25 | 20 | 15 | 10 | 5 | 0 | 35.4 | 30.4 | 25.5 | 20.6 | 15.8 | 11.2 | 7.1 | 5 | 36.4 | 31.6 | 26.9 | 22.4 | 18 | 14.1 | 11.2 | 10 | 38.1 | 33.5 | 29.2 | 25 | 21.2 | 18 | 15.8 | 15 |
| gB1 | 5 | 7.1 | 11.2 | 15.8 | 20.6 | 25.5 | 30.4 | 35.4 | 0 | 5 | 10 | 15 | 20 | 25 | 30 | 35 | 5 | 7.1 | 11.2 | 15.8 | 20.6 | 25.5 | 30.4 | 35.4 | 10 | 11.2 | 14.1 | 18 | 22.4 | 26.9 | 31.6 | 36.4 |
| gB2 | 7.1 | 5 | 7.1 | 11.2 | 15.8 | 20.6 | 25.5 | 30.4 | 5 | 0 | 5 | 10 | 15 | 20 | 25 | 30 | 7.1 | 5 | 7.1 | 11.2 | 15.8 | 20.6 | 25.5 | 30.4 | 11.2 | 10 | 11.2 | 14.1 | 18 | 22.4 | 26.9 | 31.6 |
| gB3 | 11.2 | 7.1 | 5 | 7.1 | 11.2 | 15.8 | 20.6 | 25.5 | 10 | 5 | 0 | 5 | 10 | 15 | 20 | 25 | 11.2 | 7.1 | 5 | 7.1 | 11.2 | 15.8 | 20.6 | 25.5 | 14.1 | 11.2 | 10 | 11.2 | 14.1 | 18 | 22.4 | 26.9 |
| gB4 | 15.8 | 11.2 | 7.1 | 5 | 7.1 | 11.2 | 15.8 | 20.6 | 15 | 10 | 5 | 0 | 5 | 10 | 15 | 20 | 15.8 | 11.2 | 7.1 | 5 | 7.1 | 11.2 | 15.8 | 20.6 | 18 | 14.1 | 11.2 | 10 | 11.2 | 14.1 | 18 | 22.4 |
| gB5 | 20.6 | 15.8 | 11.2 | 7.1 | 5 | 7.1 | 11.2 | 15.8 | 20 | 15 | 10 | 5 | 0 | 5 | 10 | 15 | 20.6 | 15.8 | 11.2 | 7.1 | 5 | 7.1 | 11.2 | 15.8 | 22.4 | 18 | 14.1 | 11.2 | 10 | 11.2 | 14.1 | 18 |
| gB6 | 25.5 | 20.6 | 15.8 | 11.2 | 7.1 | 5 | 7.1 | 11.2 | 25 | 20 | 15 | 10 | 5 | 0 | 5 | 10 | 25.5 | 20.6 | 15.8 | 11.2 | 7.1 | 5 | 7.1 | 11.2 | 26.9 | 22.4 | 18 | 14.1 | 11.2 | 10 | 11.2 | 14.1 |
| gB7 | 30.4 | 25.5 | 20.6 | 15.8 | 11.2 | 7.1 | 5 | 7.1 | 30 | 25 | 20 | 15 | 10 | 5 | 0 | 5 | 30.4 | 25.5 | 20.6 | 15.8 | 11.2 | 7.1 | 5 | 7.1 | 31.6 | 26.9 | 22.4 | 18 | 14.1 | 11.2 | 10 | 11.2 |
| gB8 | 35.4 | 30.4 | 25.5 | 20.6 | 15.8 | 11.2 | 7.1 | 5 | 35 | 30 | 25 | 20 | 15 | 10 | 5 | 0 | 35.4 | 30.4 | 25.5 | 20.6 | 15.8 | 11.2 | 7.1 | 5 | 36.4 | 31.6 | 26.9 | 22.4 | 18 | 14.1 | 11.2 | 10 |
| gC1 | 10 | 11.2 | 14.1 | 18 | 22.4 | 26.9 | 31.6 | 36.4 | 5 | 7.1 | 11.2 | 15.8 | 20.6 | 25.5 | 30.4 | 35.4 | 0 | 5 | 10 | 15 | 20 | 25 | 30 | 35 | 5 | 7.1 | 11.2 | 15.8 | 20.6 | 25.5 | 30.4 | 35.4 |
| gC2 | 11.2 | 10 | 11.2 | 14.1 | 18 | 22.4 | 26.9 | 31.6 | 7.1 | 5 | 7.1 | 11.2 | 15.8 | 20.6 | 25.5 | 30.4 | 5 | 0 | 5 | 10 | 15 | 20 | 25 | 30 | 7.1 | 5 | 7.1 | 11.2 | 15.8 | 20.6 | 25.5 | 30.4 |
| gC3 | 14.1 | 11.2 | 10 | 11.2 | 14.1 | 18 | 22.4 | 26.9 | 11.2 | 7.1 | 5 | 7.1 | 11.2 | 15.8 | 20.6 | 25.5 | 10 | 5 | 0 | 5 | 10 | 15 | 20 | 25 | 11.2 | 7.1 | 5 | 7.1 | 11.2 | 15.8 | 20.6 | 25.5 |
| gC4 | 18 | 14.1 | 11.2 | 10 | 11.2 | 14.1 | 18 | 22.4 | 15.8 | 11.2 | 7.1 | 5 | 7.1 | 11.2 | 15.8 | 20.6 | 15 | 10 | 5 | 0 | 5 | 10 | 15 | 20 | 15.8 | 11.2 | 7.1 | 5 | 7.1 | 11.2 | 15.8 | 20.6 |
| gC5 | 22.4 | 18 | 14.1 | 11.2 | 10 | 11.2 | 14.1 | 18 | 20.6 | 15.8 | 11.2 | 7.1 | 5 | 7.1 | 11.2 | 15.8 | 20 | 15 | 10 | 5 | 0 | 5 | 10 | 15 | 20.6 | 15.8 | 11.2 | 7.1 | 5 | 7.1 | 11.2 | 15.8 |
| gC6 | 26.9 | 22.4 | 18 | 14.1 | 11.2 | 10 | 11.2 | 14.1 | 25.5 | 20.6 | 15.8 | 11.2 | 7.1 | 5 | 7.1 | 11.2 | 25 | 20 | 15 | 10 | 5 | 0 | 5 | 10 | 25.5 | 20.6 | 15.8 | 11.2 | 7.1 | 5 | 7.1 | 11.2 |
| gC7 | 31.6 | 26.9 | 22.4 | 18 | 14.1 | 11.2 | 10 | 11.2 | 30.4 | 25.5 | 20.6 | 15.8 | 11.2 | 7.1 | 5 | 7.1 | 30 | 25 | 20 | 15 | 10 | 5 | 0 | 5 | 30.4 | 25.5 | 20.6 | 15.8 | 11.2 | 7.1 | 5 | 7.1 |
| gC8 | 36.4 | 31.6 | 26.9 | 22.4 | 18 | 14.1 | 11.2 | 10 | 35.4 | 30.4 | 25.5 | 20.6 | 15.8 | 11.2 | 7.1 | 5 | 35 | 30 | 25 | 20 | 15 | 10 | 5 | 0 | 35.4 | 30.4 | 25.5 | 20.6 | 15.8 | 11.2 | 7.1 | 5 |
| gD1 | 15 | 15.8 | 18 | 21.2 | 25 | 29.2 | 33.5 | 38.1 | 10 | 11.2 | 14.1 | 18 | 22.4 | 26.9 | 31.6 | 36.4 | 5 | 7.1 | 11.2 | 15.8 | 20.6 | 25.5 | 30.4 | 35.4 | 0 | 5 | 10 | 15 | 20 | 25 | 30 | 35 |
| gD2 | 15.8 | 15 | 15.8 | 18 | 21.2 | 25 | 29.2 | 33.5 | 11.2 | 10 | 11.2 | 14.1 | 18 | 22.4 | 26.9 | 31.6 | 7.1 | 5 | 7.1 | 11.2 | 15.8 | 20.6 | 25.5 | 30.4 | 5 | 0 | 5 | 10 | 15 | 20 | 25 | 30 |
| gD3 | 18 | 15.8 | 15 | 15.8 | 18 | 21.2 | 25 | 29.2 | 14.1 | 11.2 | 10 | 11.2 | 14.1 | 18 | 22.4 | 26.9 | 11.2 | 7.1 | 5 | 7.1 | 11.2 | 15.8 | 20.6 | 25.5 | 10 | 5 | 0 | 5 | 10 | 15 | 20 | 25 |
| gD4 | 21.2 | 18 | 15.8 | 15 | 15.8 | 18 | 21.2 | 25 | 18 | 14.1 | 11.2 | 10 | 11.2 | 14.1 | 18 | 22.4 | 15.8 | 11.2 | 7.1 | 5 | 7.1 | 11.2 | 15.8 | 20.6 | 15 | 10 | 5 | 0 | 5 | 10 | 15 | 20 |
| gD5 | 25 | 21.2 | 18 | 15.8 | 15 | 15.8 | 18 | 21.2 | 22.4 | 18 | 14.1 | 11.2 | 10 | 11.2 | 14.1 | 18 | 20.6 | 15.8 | 11.2 | 7.1 | 5 | 7.1 | 11.2 | 15.8 | 20 | 15 | 10 | 5 | 0 | 5 | 10 | 15 |
| gD6 | 29.2 | 25 | 21.2 | 18 | 15.8 | 15 | 15.8 | 18 | 26.9 | 22.4 | 18 | 14.1 | 11.2 | 10 | 11.2 | 14.1 | 25.5 | 20.6 | 15.8 | 11.2 | 7.1 | 5 | 7.1 | 11.2 | 25 | 20 | 15 | 10 | 5 | 0 | 5 | 10 |
| gD7 | 33.5 | 29.2 | 25 | 21.2 | 18 | 15.8 | 15 | 15.8 | 31.6 | 26.9 | 22.4 | 18 | 14.1 | 11.2 | 10 | 11.2 | 30.4 | 25.5 | 20.6 | 15.8 | 11.2 | 7.1 | 5 | 7.1 | 30 | 25 | 20 | 15 | 10 | 5 | 0 | 5 |
| gD8 | 38.1 | 33.5 | 29.2 | 25 | 21.2 | 18 | 15.8 | 15 | 36.4 | 31.6 | 26.9 | 22.4 | 18 | 14.1 | 11.2 | 10 | 35.4 | 30.4 | 25.5 | 20.6 | 15.8 | 11.2 | 7.1 | 5 | 35 | 30 | 25 | 20 | 15 | 10 | 5 | 0 |
| gE1 | 20 | 20.6 | 22.4 | 25 | 28.3 | 32 | 36.1 | 40.3 | 15 | 15.8 | 18 | 21.2 | 25 | 29.2 | 33.5 | 38.1 | 10 | 11.2 | 14.1 | 18 | 22.4 | 26.9 | 31.6 | 36.4 | 5 | 7.1 | 11.2 | 15.8 | 20.6 | 25.5 | 30.4 | 35.4 |
| gE2 | 20.6 | 20 | 20.6 | 22.4 | 25 | 28.3 | 32 | 36.1 | 15.8 | 15 | 15.8 | 18 | 21.2 | 25 | 29.2 | 33.5 | 11.2 | 10 | 11.2 | 14.1 | 18 | 22.4 | 26.9 | 31.6 | 7.1 | 5 | 7.1 | 11.2 | 15.8 | 20.6 | 25.5 | 30.4 |
| gE3 | 22.4 | 20.6 | 20 | 20.6 | 22.4 | 25 | 28.3 | 32 | 18 | 15.8 | 15 | 15.8 | 18 | 21.2 | 25 | 29.2 | 14.1 | 11.2 | 10 | 11.2 | 14.1 | 18 | 22.4 | 26.9 | 11.2 | 7.1 | 5 | 7.1 | 11.2 | 15.8 | 20.6 | 25.5 |
| gE4 | 25 | 22.4 | 20.6 | 20 | 20.6 | 22.4 | 25 | 28.3 | 21.2 | 18 | 15.8 | 15 | 15.8 | 18 | 21.2 | 25 | 18 | 14.1 | 11.2 | 10 | 11.2 | 14.1 | 18 | 22.4 | 15.8 | 11.2 | 7.1 | 5 | 7.1 | 11.2 | 15.8 | 20.6 |
| gE5 | 28.3 | 25 | 22.4 | 20.6 | 20 | 20.6 | 22.4 | 25 | 25 | 21.2 | 18 | 15.8 | 15 | 15.8 | 18 | 21.2 | 22.4 | 18 | 14.1 | 11.2 | 10 | 11.2 | 14.1 | 18 | 20.6 | 15.8 | 11.2 | 7.1 | 5 | 7.1 | 11.2 | 15.8 |
| gE6 | 32 | 28.3 | 25 | 22.4 | 20.6 | 20 | 20.6 | 22.4 | 29.2 | 25 | 21.2 | 18 | 15.8 | 15 | 15.8 | 18 | 26.9 | 22.4 | 18 | 14.1 | 11.2 | 10 | 11.2 | 14.1 | 25.5 | 20.6 | 15.8 | 11.2 | 7.1 | 5 | 7.1 | 11.2 |
| gE7 | 36.1 | 32 | 28.3 | 25 | 22.4 | 20.6 | 20 | 20.6 | 33.5 | 29.2 | 25 | 21.2 | 18 | 15.8 | 15 | 15.8 | 31.6 | 26.9 | 22.4 | 18 | 14.1 | 11.2 | 10 | 11.2 | 30.4 | 25.5 | 20.6 | 15.8 | 11.2 | 7.1 | 5 | 7.1 |
| gE8 | 40.3 | 36.1 | 32 | 28.3 | 25 | 22.4 | 20.6 | 20 | 38.1 | 33.5 | 29.2 | 25 | 21.2 | 18 | 15.8 | 15 | 36.4 | 31.6 | 26.9 | 22.4 | 18 | 14.1 | 11.2 | 10 | 35.4 | 30.4 | 25.5 | 20.6 | 15.8 | 11.2 | 7.1 | 5 |
| gF1 | 25 | 25.5 | 26.9 | 29.2 | 32 | 35.4 | 39.1 | 43 | 20 | 20.6 | 22.4 | 25 | 28.3 | 32 | 36.1 | 40.3 | 15 | 15.8 | 18 | 21.2 | 25 | 29.2 | 33.5 | 38.1 | 10 | 11.2 | 14.1 | 18 | 22.4 | 26.9 | 31.6 | 36.4 |
| gF2 | 25.5 | 25 | 25.5 | 26.9 | 29.2 | 32 | 35.4 | 39.1 | 20.6 | 20 | 20.6 | 22.4 | 25 | 28.3 | 32 | 36.1 | 15.8 | 15 | 15.8 | 18 | 21.2 | 25 | 29.2 | 33.5 | 11.2 | 10 | 11.2 | 14.1 | 18 | 22.4 | 26.9 | 31.6 |
| gF3 | 26.9 | 25.5 | 25 | 25.5 | 26.9 | 29.2 | 32 | 35.4 | 22.4 | 20.6 | 20 | 20.6 | 22.4 | 25 | 28.3 | 32 | 18 | 15.8 | 15 | 15.8 | 18 | 21.2 | 25 | 29.2 | 14.1 | 11.2 | 10 | 11.2 | 14.1 | 18 | 22.4 | 26.9 |
| gF4 | 29.2 | 26.9 | 25.5 | 25 | 25.5 | 26.9 | 29.2 | 32 | 25 | 22.4 | 20.6 | 20 | 20.6 | 22.4 | 25 | 28.3 | 21.2 | 18 | 15.8 | 15 | 15.8 | 18 | 21.2 | 25 | 18 | 14.1 | 11.2 | 10 | 11.2 | 14.1 | 18 | 22.4 |
| gF5 | 32 | 29.2 | 26.9 | 25.5 | 25 | 25.5 | 26.9 | 29.2 | 28.3 | 25 | 22.4 | 20.6 | 20 | 20.6 | 22.4 | 25 | 25 | 21.2 | 18 | 15.8 | 15 | 15.8 | 18 | 21.2 | 22.4 | 18 | 14.1 | 11.2 | 10 | 11.2 | 14.1 | 18 |
| gF6 | 35.4 | 32 | 29.2 | 26.9 | 25.5 | 25 | 25.5 | 26.9 | 32 | 28.3 | 25 | 22.4 | 20.6 | 20 | 20.6 | 22.4 | 29.2 | 25 | 21.2 | 18 | 15.8 | 15 | 15.8 | 18 | 26.9 | 22.4 | 18 | 14.1 | 11.2 | 10 | 11.2 | 14.1 |
| gF7 | 39.1 | 35.4 | 32 | 29.2 | 26.9 | 25.5 | 25 | 25.5 | 36.1 | 32 | 28.3 | 25 | 22.4 | 20.6 | 20 | 20.6 | 33.5 | 29.2 | 25 | 21.2 | 18 | 15.8 | 15 | 15.8 | 31.6 | 26.9 | 22.4 | 18 | 14.1 | 11.2 | 10 | 11.2 |
| gF8 | 43 | 39.1 | 35.4 | 32 | 29.2 | 26.9 | 25.5 | 25 | 40.3 | 36.1 | 32 | 28.3 | 25 | 22.4 | 20.6 | 20 | 38.1 | 33.5 | 29.2 | 25 | 21.2 | 18 | 15.8 | 15 | 36.4 | 31.6 | 26.9 | 22.4 | 18 | 14.1 | 11.2 | 10 |
| gG1 | 30 | 30.4 | 31.6 | 33.5 | 36.1 | 39.1 | 42.4 | 46.1 | 25 | 25.5 | 26.9 | 29.2 | 32 | 35.4 | 39.1 | 43 | 20 | 20.6 | 22.4 | 25 | 28.3 | 32 | 36.1 | 40.3 | 15 | 15.8 | 18 | 21.2 | 25 | 29.2 | 33.5 | 38.1 |
| gG2 | 30.4 | 30 | 30.4 | 31.6 | 33.5 | 36.1 | 39.1 | 42.4 | 25.5 | 25 | 25.5 | 26.9 | 29.2 | 32 | 35.4 | 39.1 | 20.6 | 20 | 20.6 | 22.4 | 25 | 28.3 | 32 | 36.1 | 15.8 | 15 | 15.8 | 18 | 21.2 | 25 | 29.2 | 33.5 |
| gG3 | 31.6 | 30.4 | 30 | 30.4 | 31.6 | 33.5 | 36.1 | 39.1 | 26.9 | 25.5 | 25 | 25.5 | 26.9 | 29.2 | 32 | 35.4 | 22.4 | 20.6 | 20 | 20.6 | 22.4 | 25 | 28.3 | 32 | 18 | 15.8 | 15 | 15.8 | 18 | 21.2 | 25 | 29.2 |
| gG4 | 33.5 | 31.6 | 30.4 | 30 | 30.4 | 31.6 | 33.5 | 36.1 | 29.2 | 26.9 | 25.5 | 25 | 25.5 | 26.9 | 29.2 | 32 | 25 | 22.4 | 20.6 | 20 | 20.6 | 22.4 | 25 | 28.3 | 21.2 | 18 | 15.8 | 15 | 15.8 | 18 | 21.2 | 25 |
| gG5 | 36.1 | 33.5 | 31.6 | 30.4 | 30 | 30.4 | 31.6 | 33.5 | 32 | 29.2 | 26.9 | 25.5 | 25 | 25.5 | 26.9 | 29.2 | 28.3 | 25 | 22.4 | 20.6 | 20 | 20.6 | 22.4 | 25 | 25 | 21.2 | 18 | 15.8 | 15 | 15.8 | 18 | 21.2 |
| gG6 | 39.1 | 36.1 | 33.5 | 31.6 | 30.4 | 30 | 30.4 | 31.6 | 35.4 | 32 | 29.2 | 26.9 | 25.5 | 25 | 25.5 | 26.9 | 32 | 28.3 | 25 | 22.4 | 20.6 | 20 | 20.6 | 22.4 | 29.2 | 25 | 21.2 | 18 | 15.8 | 15 | 15.8 | 18 |
| gG7 | 42.4 | 39.1 | 36.1 | 33.5 | 31.6 | 30.4 | 30 | 30.4 | 39.1 | 35.4 | 32 | 29.2 | 26.9 | 25.5 | 25 | 25.5 | 36.1 | 32 | 28.3 | 25 | 22.4 | 20.6 | 20 | 20.6 | 33.5 | 29.2 | 25 | 21.2 | 18 | 15.8 | 15 | 15.8 |
| gG8 | 46.1 | 42.4 | 39.1 | 36.1 | 33.5 | 31.6 | 30.4 | 30 | 43 | 39.1 | 35.4 | 32 | 29.2 | 26.9 | 25.5 | 25 | 40.3 | 36.1 | 32 | 28.3 | 25 | 22.4 | 20.6 | 20 | 38.1 | 33.5 | 29.2 | 25 | 21.2 | 18 | 15.8 | 15 |
| gH1 | 35 | 35.4 | 36.4 | 38.1 | 40.3 | 43 | 46.1 | 49.5 | 30 | 30.4 | 31.6 | 33.5 | 36.1 | 39.1 | 42.4 | 46.1 | 25 | 25.5 | 26.9 | 29.2 | 32 | 35.4 | 39.1 | 43 | 20 | 20.6 | 22.4 | 25 | 28.3 | 32 | 36.1 | 40.3 |
| gH2 | 35.4 | 35 | 35.4 | 36.4 | 38.1 | 40.3 | 43 | 46.1 | 30.4 | 30 | 30.4 | 31.6 | 33.5 | 36.1 | 39.1 | 42.4 | 25.5 | 25 | 25.5 | 26.9 | 29.2 | 32 | 35.4 | 39.1 | 20.6 | 20 | 20.6 | 22.4 | 25 | 28.3 | 32 | 36.1 |
| gH3 | 36.4 | 35.4 | 35 | 35.4 | 36.4 | 38.1 | 40.3 | 43 | 31.6 | 30.4 | 30 | 30.4 | 31.6 | 33.5 | 36.1 | 39.1 | 26.9 | 25.5 | 25 | 25.5 | 26.9 | 29.2 | 32 | 35.4 | 22.4 | 20.6 | 20 | 20.6 | 22.4 | 25 | 28.3 | 32 |
| gH4 | 38.1 | 36.4 | 35.4 | 35 | 35.4 | 36.4 | 38.1 | 40.3 | 33.5 | 31.6 | 30.4 | 30 | 30.4 | 31.6 | 33.5 | 36.1 | 29.2 | 26.9 | 25.5 | 25 | 25.5 | 26.9 | 29.2 | 32 | 25 | 22.4 | 20.6 | 20 | 20.6 | 22.4 | 25 | 28.3 |
| gH5 | 40.3 | 38.1 | 36.4 | 35.4 | 35 | 35.4 | 36.4 | 38.1 | 36.1 | 33.5 | 31.6 | 30.4 | 30 | 30.4 | 31.6 | 33.5 | 32 | 29.2 | 26.9 | 25.5 | 25 | 25.5 | 26.9 | 29.2 | 28.3 | 25 | 22.4 | 20.6 | 20 | 20.6 | 22.4 | 25 |
| gH6 | 43 | 40.3 | 38.1 | 36.4 | 35.4 | 35 | 35.4 | 36.4 | 39.1 | 36.1 | 33.5 | 31.6 | 30.4 | 30 | 30.4 | 31.6 | 35.4 | 32 | 29.2 | 26.9 | 25.5 | 25 | 25.5 | 26.9 | 32 | 28.3 | 25 | 22.4 | 20.6 | 20 | 20.6 | 22.4 |
| gH7 | 46.1 | 43 | 40.3 | 38.1 | 36.4 | 35.4 | 35 | 35.4 | 42.4 | 39.1 | 36.1 | 33.5 | 31.6 | 30.4 | 30 | 30.4 | 39.1 | 35.4 | 32 | 29.2 | 26.9 | 25.5 | 25 | 25.5 | 36.1 | 32 | 28.3 | 25 | 22.4 | 20.6 | 20 | 20.6 |
| gH8 | 49.5 | 46.1 | 43 | 40.3 | 38.1 | 36.4 | 35.4 | 35 | 46.1 | 42.4 | 39.1 | 36.1 | 33.5 | 31.6 | 30.4 | 30 | 43 | 39.1 | 35.4 | 32 | 29.2 | 26.9 | 25.5 | 25 | 40.3 | 36.1 | 32 | 28.3 | 25 | 22.4 | 20.6 | 20 |

***range*** $_{m'm''}$ ***(continued)***

| | gE1 | gE2 | gE3 | gE4 | gE5 | gE6 | gE7 | gE8 | gF1 | gF2 | gF3 | gF4 | gF5 | gF6 | gF7 | gF8 | gG1 | gG2 | gG3 | gG4 | gG5 | gG6 | gG7 | gG8 | gH1 | gH2 | gH3 | gH4 | gH5 | gH6 | gH7 | gH8 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| gA1 | 20 | 20.6 | 22.4 | 25 | 28.3 | 32 | 36.1 | 40.3 | 25 | 25.5 | 26.9 | 29.2 | 32 | 35.4 | 39.1 | 43 | 30 | 30.4 | 31.6 | 33.5 | 36.1 | 39.1 | 42.4 | 46.1 | 35 | 35.4 | 36.4 | 38.1 | 40.3 | 43 | 46.1 | 49.5 |
| gA2 | 20.6 | 20 | 20.6 | 22.4 | 25 | 28.3 | 32 | 36.1 | 25.5 | 25 | 25.5 | 26.9 | 29.2 | 32 | 35.4 | 39.1 | 30.4 | 30 | 30.4 | 31.6 | 33.5 | 36.1 | 39.1 | 42.4 | 35.4 | 35 | 35.4 | 36.4 | 38.1 | 40.3 | 43 | 46.1 |
| gA3 | 22.4 | 20.6 | 20 | 20.6 | 22.4 | 25 | 28.3 | 32 | 26.9 | 25.5 | 25 | 25.5 | 26.9 | 29.2 | 32 | 35.4 | 31.6 | 30.4 | 30 | 30.4 | 31.6 | 33.5 | 36.1 | 39.1 | 36.4 | 35.4 | 35 | 35.4 | 36.4 | 38.1 | 40.3 | 43 |
| gA4 | 25 | 22.4 | 20.6 | 20 | 20.6 | 22.4 | 25 | 28.3 | 29.2 | 26.9 | 25.5 | 25 | 25.5 | 26.9 | 29.2 | 32 | 33.5 | 31.6 | 30.4 | 30 | 30.4 | 31.6 | 33.5 | 36.1 | 38.1 | 36.4 | 35.4 | 35 | 35.4 | 36.4 | 38.1 | 40.3 |
| gA5 | 28.3 | 25 | 22.4 | 20.6 | 20 | 20.6 | 22.4 | 25 | 32 | 29.2 | 26.9 | 25.5 | 25 | 25.5 | 26.9 | 29.2 | 36.1 | 33.5 | 31.6 | 30.4 | 30 | 30.4 | 31.6 | 33.5 | 40.3 | 38.1 | 36.4 | 35.4 | 35 | 35.4 | 36.4 | 38.1 |
| gA6 | 32 | 28.3 | 25 | 22.4 | 20.6 | 20 | 20.6 | 22.4 | 35.4 | 32 | 29.2 | 26.9 | 25.5 | 25 | 25.5 | 26.9 | 39.1 | 36.1 | 33.5 | 31.6 | 30.4 | 30 | 30.4 | 31.6 | 43 | 40.3 | 38.1 | 36.4 | 35.4 | 35 | 35.4 | 36.4 |
| gA7 | 36.1 | 32 | 28.3 | 25 | 22.4 | 20.6 | 20 | 20.6 | 39.1 | 35.4 | 32 | 29.2 | 26.9 | 25.5 | 25 | 25.5 | 42.4 | 39.1 | 36.1 | 33.5 | 31.6 | 30.4 | 30 | 30.4 | 46.1 | 43 | 40.3 | 38.1 | 36.4 | 35.4 | 35 | 35.4 |
| gA8 | 40.3 | 36.1 | 32 | 28.3 | 25 | 22.4 | 20.6 | 20 | 43 | 39.1 | 35.4 | 32 | 29.2 | 26.9 | 25.5 | 25 | 46.1 | 42.4 | 39.1 | 36.1 | 33.5 | 31.6 | 30.4 | 30 | 49.5 | 46.1 | 43 | 40.3 | 38.1 | 36.4 | 35.4 | 35 |
| gB1 | 15 | 15.8 | 18 | 21.2 | 25 | 29.2 | 33.5 | 38.1 | 20 | 20.6 | 22.4 | 25 | 28.3 | 32 | 36.1 | 40.3 | 25 | 25.5 | 26.9 | 29.2 | 32 | 35.4 | 39.1 | 43 | 30 | 30.4 | 31.6 | 33.5 | 36.1 | 39.1 | 42.4 | 46.1 |
| gB2 | 15.8 | 15 | 15.8 | 18 | 21.2 | 25 | 29.2 | 33.5 | 20.6 | 20 | 20.6 | 22.4 | 25 | 28.3 | 32 | 36.1 | 25.5 | 25 | 25.5 | 26.9 | 29.2 | 32 | 35.4 | 39.1 | 30.4 | 30 | 30.4 | 31.6 | 33.5 | 36.1 | 39.1 | 42.4 |
| gB3 | 18 | 15.8 | 15 | 15.8 | 18 | 21.2 | 25 | 29.2 | 22.4 | 20.6 | 20 | 20.6 | 22.4 | 25 | 28.3 | 32 | 26.9 | 25.5 | 25 | 25.5 | 26.9 | 29.2 | 32 | 35.4 | 31.6 | 30.4 | 30 | 30.4 | 31.6 | 33.5 | 36.1 | 39.1 |
| gB4 | 21.2 | 18 | 15.8 | 15 | 15.8 | 18 | 21.2 | 25 | 25 | 22.4 | 20.6 | 20 | 20.6 | 22.4 | 25 | 28.3 | 29.2 | 26.9 | 25.5 | 25 | 25.5 | 26.9 | 29.2 | 32 | 33.5 | 31.6 | 30.4 | 30 | 30.4 | 31.6 | 33.5 | 36.1 |
| gB5 | 25 | 21.2 | 18 | 15.8 | 15 | 15.8 | 18 | 21.2 | 28.3 | 25 | 22.4 | 20.6 | 20 | 20.6 | 22.4 | 25 | 32 | 29.2 | 26.9 | 25.5 | 25 | 25.5 | 26.9 | 29.2 | 36.1 | 33.5 | 31.6 | 30.4 | 30 | 30.4 | 31.6 | 33.5 |
| gB6 | 29.2 | 25 | 21.2 | 18 | 15.8 | 15 | 15.8 | 18 | 32 | 28.3 | 25 | 22.4 | 20.6 | 20 | 20.6 | 22.4 | 35.4 | 32 | 29.2 | 26.9 | 25.5 | 25 | 25.5 | 26.9 | 39.1 | 36.1 | 33.5 | 31.6 | 30.4 | 30 | 30.4 | 31.6 |
| gB7 | 33.5 | 29.2 | 25 | 21.2 | 18 | 15.8 | 15 | 15.8 | 36.1 | 32 | 28.3 | 25 | 22.4 | 20.6 | 20 | 20.6 | 39.1 | 35.4 | 32 | 29.2 | 26.9 | 25.5 | 25 | 25.5 | 42.4 | 39.1 | 36.1 | 33.5 | 31.6 | 30.4 | 30 | 30.4 |
| gB8 | 38.1 | 33.5 | 29.2 | 25 | 21.2 | 18 | 15.8 | 15 | 40.3 | 36.1 | 32 | 28.3 | 25 | 22.4 | 20.6 | 20 | 43 | 39.1 | 35.4 | 32 | 29.2 | 26.9 | 25.5 | 25 | 46.1 | 42.4 | 39.1 | 36.1 | 33.5 | 31.6 | 30.4 | 30 |
| gC1 | 10 | 11.2 | 14.1 | 18 | 22.4 | 26.9 | 31.6 | 36.4 | 15 | 15.8 | 18 | 21.2 | 25 | 29.2 | 33.5 | 38.1 | 20 | 20.6 | 22.4 | 25 | 28.3 | 32 | 36.1 | 40.3 | 25 | 25.5 | 26.9 | 29.2 | 32 | 35.4 | 39.1 | 43 |
| gC2 | 11.2 | 10 | 11.2 | 14.1 | 18 | 22.4 | 26.9 | 31.6 | 15.8 | 15 | 15.8 | 18 | 21.2 | 25 | 29.2 | 33.5 | 20.6 | 20 | 20.6 | 22.4 | 25 | 28.3 | 32 | 36.1 | 25.5 | 25 | 25.5 | 26.9 | 29.2 | 32 | 35.4 | 39.1 |
| gC3 | 14.1 | 11.2 | 10 | 11.2 | 14.1 | 18 | 22.4 | 26.9 | 18 | 15.8 | 15 | 15.8 | 18 | 21.2 | 25 | 29.2 | 22.4 | 20.6 | 20 | 20.6 | 22.4 | 25 | 28.3 | 32 | 26.9 | 25.5 | 25 | 25.5 | 26.9 | 29.2 | 32 | 35.4 |
| gC4 | 18 | 14.1 | 11.2 | 10 | 11.2 | 14.1 | 18 | 22.4 | 21.2 | 18 | 15.8 | 15 | 15.8 | 18 | 21.2 | 25 | 25 | 22.4 | 20.6 | 20 | 20.6 | 22.4 | 25 | 28.3 | 29.2 | 26.9 | 25.5 | 25 | 25.5 | 26.9 | 29.2 | 32 |
| gC5 | 22.4 | 18 | 14.1 | 11.2 | 10 | 11.2 | 14.1 | 18 | 25 | 21.2 | 18 | 15.8 | 15 | 15.8 | 18 | 21.2 | 28.3 | 25 | 22.4 | 20.6 | 20 | 20.6 | 22.4 | 25 | 32 | 29.2 | 26.9 | 25.5 | 25 | 25.5 | 26.9 | 29.2 |
| gC6 | 26.9 | 22.4 | 18 | 14.1 | 11.2 | 10 | 11.2 | 14.1 | 29.2 | 25 | 21.2 | 18 | 15.8 | 15 | 15.8 | 18 | 32 | 28.3 | 25 | 22.4 | 20.6 | 20 | 20.6 | 22.4 | 35.4 | 32 | 29.2 | 26.9 | 25.5 | 25 | 25.5 | 26.9 |
| gC7 | 31.6 | 26.9 | 22.4 | 18 | 14.1 | 11.2 | 10 | 11.2 | 33.5 | 29.2 | 25 | 21.2 | 18 | 15.8 | 15 | 15.8 | 36.1 | 32 | 28.3 | 25 | 22.4 | 20.6 | 20 | 20.6 | 39.1 | 35.4 | 32 | 29.2 | 26.9 | 25.5 | 25 | 25.5 |
| gC8 | 36.4 | 31.6 | 26.9 | 22.4 | 18 | 14.1 | 11.2 | 10 | 38.1 | 33.5 | 29.2 | 25 | 21.2 | 18 | 15.8 | 15 | 40.3 | 36.1 | 32 | 28.3 | 25 | 22.4 | 20.6 | 20 | 43 | 39.1 | 35.4 | 32 | 29.2 | 26.9 | 25.5 | 25 |
| gD1 | 5 | 7.1 | 11.2 | 15.8 | 20.6 | 25.5 | 30.4 | 35.4 | 10 | 11.2 | 14.1 | 18 | 22.4 | 26.9 | 31.6 | 36.4 | 15 | 15.8 | 18 | 21.2 | 25 | 29.2 | 33.5 | 38.1 | 20 | 20.6 | 22.4 | 25 | 28.3 | 32 | 36.1 | 40.3 |
| gD2 | 7.1 | 5 | 7.1 | 11.2 | 15.8 | 20.6 | 25.5 | 30.4 | 11.2 | 10 | 11.2 | 14.1 | 18 | 22.4 | 26.9 | 31.6 | 15.8 | 15 | 15.8 | 18 | 21.2 | 25 | 29.2 | 33.5 | 20.6 | 20 | 20.6 | 22.4 | 25 | 28.3 | 32 | 36.1 |
| gD3 | 11.2 | 7.1 | 5 | 7.1 | 11.2 | 15.8 | 20.6 | 25.5 | 14.1 | 11.2 | 10 | 11.2 | 14.1 | 18 | 22.4 | 26.9 | 18 | 15.8 | 15 | 15.8 | 18 | 21.2 | 25 | 29.2 | 22.4 | 20.6 | 20 | 20.6 | 22.4 | 25 | 28.3 | 32 |
| gD4 | 15.8 | 11.2 | 7.1 | 5 | 7.1 | 11.2 | 15.8 | 20.6 | 18 | 14.1 | 11.2 | 10 | 11.2 | 14.1 | 18 | 22.4 | 21.2 | 18 | 15.8 | 15 | 15.8 | 18 | 21.2 | 25 | 25 | 22.4 | 20.6 | 20 | 20.6 | 22.4 | 25 | 28.3 |
| gD5 | 20.6 | 15.8 | 11.2 | 7.1 | 5 | 7.1 | 11.2 | 15.8 | 22.4 | 18 | 14.1 | 11.2 | 10 | 11.2 | 14.1 | 18 | 25 | 21.2 | 18 | 15.8 | 15 | 15.8 | 18 | 21.2 | 28.3 | 25 | 22.4 | 20.6 | 20 | 20.6 | 22.4 | 25 |
| gD6 | 25.5 | 20.6 | 15.8 | 11.2 | 7.1 | 5 | 7.1 | 11.2 | 26.9 | 22.4 | 18 | 14.1 | 11.2 | 10 | 11.2 | 14.1 | 29.2 | 25 | 21.2 | 18 | 15.8 | 15 | 15.8 | 18 | 32 | 28.3 | 25 | 22.4 | 20.6 | 20 | 20.6 | 22.4 |
| gD7 | 30.4 | 25.5 | 20.6 | 15.8 | 11.2 | 7.1 | 5 | 7.1 | 31.6 | 26.9 | 22.4 | 18 | 14.1 | 11.2 | 10 | 11.2 | 33.5 | 29.2 | 25 | 21.2 | 18 | 15.8 | 15 | 15.8 | 36.1 | 32 | 28.3 | 25 | 22.4 | 20.6 | 20 | 20.6 |
| gD8 | 35.4 | 30.4 | 25.5 | 20.6 | 15.8 | 11.2 | 7.1 | 5 | 36.4 | 31.6 | 26.9 | 22.4 | 18 | 14.1 | 11.2 | 10 | 38.1 | 33.5 | 29.2 | 25 | 21.2 | 18 | 15.8 | 15 | 40.3 | 36.1 | 32 | 28.3 | 25 | 22.4 | 20.6 | 20 |
| gE1 | 0 | 5 | 10 | 15 | 20 | 25 | 30 | 35 | 5 | 7.1 | 11.2 | 15.8 | 20.6 | 25.5 | 30.4 | 35.4 | 10 | 11.2 | 14.1 | 18 | 22.4 | 26.9 | 31.6 | 36.4 | 15 | 15.8 | 18 | 21.2 | 25 | 29.2 | 33.5 | 38.1 |
| gE2 | 5 | 0 | 5 | 10 | 15 | 20 | 25 | 30 | 7.1 | 5 | 7.1 | 11.2 | 15.8 | 20.6 | 25.5 | 30.4 | 11.2 | 10 | 11.2 | 14.1 | 18 | 22.4 | 26.9 | 31.6 | 15.8 | 15 | 15.8 | 18 | 21.2 | 25 | 29.2 | 33.5 |
| gE3 | 10 | 5 | 0 | 5 | 10 | 15 | 20 | 25 | 11.2 | 7.1 | 5 | 7.1 | 11.2 | 15.8 | 20.6 | 25.5 | 14.1 | 11.2 | 10 | 11.2 | 14.1 | 18 | 22.4 | 26.9 | 18 | 15.8 | 15 | 15.8 | 18 | 21.2 | 25 | 29.2 |
| gE4 | 15 | 10 | 5 | 0 | 5 | 10 | 15 | 20 | 15.8 | 11.2 | 7.1 | 5 | 7.1 | 11.2 | 15.8 | 20.6 | 18 | 14.1 | 11.2 | 10 | 11.2 | 14.1 | 18 | 22.4 | 21.2 | 18 | 15.8 | 15 | 15.8 | 18 | 21.2 | 25 |
| gE5 | 20 | 15 | 10 | 5 | 0 | 5 | 10 | 15 | 20.6 | 15.8 | 11.2 | 7.1 | 5 | 7.1 | 11.2 | 15.8 | 22.4 | 18 | 14.1 | 11.2 | 10 | 11.2 | 14.1 | 18 | 25 | 21.2 | 18 | 15.8 | 15 | 15.8 | 18 | 21.2 |
| gE6 | 25 | 20 | 15 | 10 | 5 | 0 | 5 | 10 | 25.5 | 20.6 | 15.8 | 11.2 | 7.1 | 5 | 7.1 | 11.2 | 26.9 | 22.4 | 18 | 14.1 | 11.2 | 10 | 11.2 | 14.1 | 29.2 | 25 | 21.2 | 18 | 15.8 | 15 | 15.8 | 18 |
| gE7 | 30 | 25 | 20 | 15 | 10 | 5 | 0 | 5 | 30.4 | 25.5 | 20.6 | 15.8 | 11.2 | 7.1 | 5 | 7.1 | 31.6 | 26.9 | 22.4 | 18 | 14.1 | 11.2 | 10 | 11.2 | 33.5 | 29.2 | 25 | 21.2 | 18 | 15.8 | 15 | 15.8 |
| gE8 | 35 | 30 | 25 | 20 | 15 | 10 | 5 | 0 | 35.4 | 30.4 | 25.5 | 20.6 | 15.8 | 11.2 | 7.1 | 5 | 36.4 | 31.6 | 26.9 | 22.4 | 18 | 14.1 | 11.2 | 10 | 38.1 | 33.5 | 29.2 | 25 | 21.2 | 18 | 15.8 | 15 |
| gF1 | 5 | 7.1 | 11.2 | 15.8 | 20.6 | 25.5 | 30.4 | 35.4 | 0 | 5 | 10 | 15 | 20 | 25 | 30 | 35 | 5 | 7.1 | 11.2 | 15.8 | 20.6 | 25.5 | 30.4 | 35.4 | 10 | 11.2 | 14.1 | 18 | 22.4 | 26.9 | 31.6 | 36.4 |
| gF2 | 7.1 | 5 | 7.1 | 11.2 | 15.8 | 20.6 | 25.5 | 30.4 | 5 | 0 | 5 | 10 | 15 | 20 | 25 | 30 | 7.1 | 5 | 7.1 | 11.2 | 15.8 | 20.6 | 25.5 | 30.4 | 11.2 | 10 | 11.2 | 14.1 | 18 | 22.4 | 26.9 | 31.6 |
| gF3 | 11.2 | 7.1 | 5 | 7.1 | 11.2 | 15.8 | 20.6 | 25.5 | 10 | 5 | 0 | 5 | 10 | 15 | 20 | 25 | 11.2 | 7.1 | 5 | 7.1 | 11.2 | 15.8 | 20.6 | 25.5 | 14.1 | 11.2 | 10 | 11.2 | 14.1 | 18 | 22.4 | 26.9 |
| gF4 | 15.8 | 11.2 | 7.1 | 5 | 7.1 | 11.2 | 15.8 | 20.6 | 15 | 10 | 5 | 0 | 5 | 10 | 15 | 20 | 15.8 | 11.2 | 7.1 | 5 | 7.1 | 11.2 | 15.8 | 20.6 | 18 | 14.1 | 11.2 | 10 | 11.2 | 14.1 | 18 | 22.4 |
| gF5 | 20.6 | 15.8 | 11.2 | 7.1 | 5 | 7.1 | 11.2 | 15.8 | 20 | 15 | 10 | 5 | 0 | 5 | 10 | 15 | 20.6 | 15.8 | 11.2 | 7.1 | 5 | 7.1 | 11.2 | 15.8 | 22.4 | 18 | 14.1 | 11.2 | 10 | 11.2 | 14.1 | 18 |
| gF6 | 25.5 | 20.6 | 15.8 | 11.2 | 7.1 | 5 | 7.1 | 11.2 | 25 | 20 | 15 | 10 | 5 | 0 | 5 | 10 | 25.5 | 20.6 | 15.8 | 11.2 | 7.1 | 5 | 7.1 | 11.2 | 26.9 | 22.4 | 18 | 14.1 | 11.2 | 10 | 11.2 | 14.1 |
| gF7 | 30.4 | 25.5 | 20.6 | 15.8 | 11.2 | 7.1 | 5 | 7.1 | 30 | 25 | 20 | 15 | 10 | 5 | 0 | 5 | 30.4 | 25.5 | 20.6 | 15.8 | 11.2 | 7.1 | 5 | 7.1 | 31.6 | 26.9 | 22.4 | 18 | 14.1 | 11.2 | 10 | 11.2 |
| gF8 | 35.4 | 30.4 | 25.5 | 20.6 | 15.8 | 11.2 | 7.1 | 5 | 35 | 30 | 25 | 20 | 15 | 10 | 5 | 0 | 35.4 | 30.4 | 25.5 | 20.6 | 15.8 | 11.2 | 7.1 | 5 | 36.4 | 31.6 | 26.9 | 22.4 | 18 | 14.1 | 11.2 | 10 |
| gG1 | 10 | 11.2 | 14.1 | 18 | 22.4 | 26.9 | 31.6 | 36.4 | 5 | 7.1 | 11.2 | 15.8 | 20.6 | 25.5 | 30.4 | 35.4 | 0 | 5 | 10 | 15 | 20 | 25 | 30 | 35 | 5 | 7.1 | 11.2 | 15.8 | 20.6 | 25.5 | 30.4 | 35.4 |
| gG2 | 11.2 | 10 | 11.2 | 14.1 | 18 | 22.4 | 26.9 | 31.6 | 7.1 | 5 | 7.1 | 11.2 | 15.8 | 20.6 | 25.5 | 30.4 | 5 | 0 | 5 | 10 | 15 | 20 | 25 | 30 | 7.1 | 5 | 7.1 | 11.2 | 15.8 | 20.6 | 25.5 | 30.4 |
| gG3 | 14.1 | 11.2 | 10 | 11.2 | 14.1 | 18 | 22.4 | 26.9 | 11.2 | 7.1 | 5 | 7.1 | 11.2 | 15.8 | 20.6 | 25.5 | 10 | 5 | 0 | 5 | 10 | 15 | 20 | 25 | 11.2 | 7.1 | 5 | 7.1 | 11.2 | 15.8 | 20.6 | 25.5 |
| gG4 | 18 | 14.1 | 11.2 | 10 | 11.2 | 14.1 | 18 | 22.4 | 15.8 | 11.2 | 7.1 | 5 | 7.1 | 11.2 | 15.8 | 20.6 | 15 | 10 | 5 | 0 | 5 | 10 | 15 | 20 | 15.8 | 11.2 | 7.1 | 5 | 7.1 | 11.2 | 15.8 | 20.6 |
| gG5 | 22.4 | 18 | 14.1 | 11.2 | 10 | 11.2 | 14.1 | 18 | 20.6 | 15.8 | 11.2 | 7.1 | 5 | 7.1 | 11.2 | 15.8 | 20 | 15 | 10 | 5 | 0 | 5 | 10 | 15 | 20.6 | 15.8 | 11.2 | 7.1 | 5 | 7.1 | 11.2 | 15.8 |
| gG6 | 26.9 | 22.4 | 18 | 14.1 | 11.2 | 10 | 11.2 | 14.1 | 25.5 | 20.6 | 15.8 | 11.2 | 7.1 | 5 | 7.1 | 11.2 | 25 | 20 | 15 | 10 | 5 | 0 | 5 | 10 | 25.5 | 20.6 | 15.8 | 11.2 | 7.1 | 5 | 7.1 | 11.2 |
| gG7 | 31.6 | 26.9 | 22.4 | 18 | 14.1 | 11.2 | 10 | 11.2 | 30.4 | 25.5 | 20.6 | 15.8 | 11.2 | 7.1 | 5 | 7.1 | 30 | 25 | 20 | 15 | 10 | 5 | 0 | 5 | 30.4 | 25.5 | 20.6 | 15.8 | 11.2 | 7.1 | 5 | 7.1 |
| gG8 | 36.4 | 31.6 | 26.9 | 22.4 | 18 | 14.1 | 11.2 | 10 | 35.4 | 30.4 | 25.5 | 20.6 | 15.8 | 11.2 | 7.1 | 5 | 35 | 30 | 25 | 20 | 15 | 10 | 5 | 0 | 35.4 | 30.4 | 25.5 | 20.6 | 15.8 | 11.2 | 7.1 | 5 |
| gH1 | 15 | 15.8 | 18 | 21.2 | 25 | 29.2 | 33.5 | 38.1 | 10 | 11.2 | 14.1 | 18 | 22.4 | 26.9 | 31.6 | 36.4 | 5 | 7.1 | 11.2 | 15.8 | 20.6 | 25.5 | 30.4 | 35.4 | 0 | 5 | 10 | 15 | 20 | 25 | 30 | 35 |
| gH2 | 15.8 | 15 | 15.8 | 18 | 21.2 | 25 | 29.2 | 33.5 | 11.2 | 10 | 11.2 | 14.1 | 18 | 22.4 | 26.9 | 31.6 | 7.1 | 5 | 7.1 | 11.2 | 15.8 | 20.6 | 25.5 | 30.4 | 5 | 0 | 5 | 10 | 15 | 20 | 25 | 30 |
| gH3 | 18 | 15.8 | 15 | 15.8 | 18 | 21.2 | 25 | 29.2 | 14.1 | 11.2 | 10 | 11.2 | 14.1 | 18 | 22.4 | 26.9 | 11.2 | 7.1 | 5 | 7.1 | 11.2 | 15.8 | 20.6 | 25.5 | 10 | 5 | 0 | 5 | 10 | 15 | 20 | 25 |
| gH4 | 21.2 | 18 | 15.8 | 15 | 15.8 | 18 | 21.2 | 25 | 18 | 14.1 | 11.2 | 10 | 11.2 | 14.1 | 18 | 22.4 | 15.8 | 11.2 | 7.1 | 5 | 7.1 | 11.2 | 15.8 | 20.6 | 15 | 10 | 5 | 0 | 5 | 10 | 15 | 20 |
| gH5 | 25 | 21.2 | 18 | 15.8 | 15 | 15.8 | 18 | 21.2 | 22.4 | 18 | 14.1 | 11.2 | 10 | 11.2 | 14.1 | 18 | 20.6 | 15.8 | 11.2 | 7.1 | 5 | 7.1 | 11.2 | 15.8 | 20 | 15 | 10 | 5 | 0 | 5 | 10 | 15 |
| gH6 | 29.2 | 25 | 21.2 | 18 | 15.8 | 15 | 15.8 | 18 | 26.9 | 22.4 | 18 | 14.1 | 11.2 | 10 | 11.2 | 14.1 | 25.5 | 20.6 | 15.8 | 11.2 | 7.1 | 5 | 7.1 | 11.2 | 25 | 20 | 15 | 10 | 5 | 0 | 5 | 10 |
| gH7 | 33.5 | 29.2 | 25 | 21.2 | 18 | 15.8 | 15 | 15.8 | 31.6 | 26.9 | 22.4 | 18 | 14.1 | 11.2 | 10 | 11.2 | 30.4 | 25.5 | 20.6 | 15.8 | 11.2 | 7.1 | 5 | 7.1 | 30 | 25 | 20 | 15 | 10 | 5 | 0 | 5 |
| gH8 | 38.1 | 33.5 | 29.2 | 25 | 21.2 | 18 | 15.8 | 15 | 36.4 | 31.6 | 26.9 | 22.4 | 18 | 14.1 | 11.2 | 10 | 35.4 | 30.4 | 25.5 | 20.6 | 15.8 | 11.2 | 7.1 | 5 | 35 | 30 | 25 | 20 | 15 | 10 | 5 | 0 |

# E. EXAMPLE FIVE: CHOKE-POINT SCENARIO

## Sets

| $P_{VIS}$ | $P_{SECRET}$ | $P_{FLEX}$ | $P_{BASE}$ | $P_{TETH}$ | $PP_{TETH}$ | $G$ | $M$ | $M_{ps}$ |
|---|---|---|---|---|---|---|---|---|
| Surf1 | (empty) | SSN1 | Surf1 | Helo1 | Surf1.Helo1 | $g^+$ | {m1,…,m198} | {Surf1.m1.active,…,Surf1.m198.active} |
| Helo1 | | | | | | gA1 | | {Helo1.m1.active,…,Helo1.m198.active} |
| | | | | | | gA2 | | {SSN1.m1.active,…,SSN1.m198.active} |
| | | | | | | gA3 (impassable) | | {SSN1.m1.passive,…,SSN1.m198.passive} |
| | | | | | | gA4 (impassable) | | |
| | | | | | | gA5 | | |
| | | | | | | gA6 | | |

| *A* | | | | | | $G$ (cont.) |
|---|---|---|---|---|---|---|
| | | | | | | gB1 |
| g+.gA1 | gB4.gB5 | gC5.gB6 | gD5.gE4 | gF1.gE1 | | gB2 |
| g+.gB1 | gB4.gC3 | gC5.gC4 | gD5.gE5 | gF1.gE2 | | gB3 |
| g+.gC1 | gB4.gC4 | gC5.gC6 | gD5.gE6 | gF1.gF2 | | gB4 |
| g+.gD1 | gB4.gC5 | gC5.gD4 | gD6.gC5 | gF2.gE1 | | gB5 |
| g+.gE1 | gB5.gA5 | gC5.gD5 | gD6.gC6 | gF2.gE2 | | gB6 |
| g+.gF1 | gB5.gA6 | gC5.gD6 | gD6.gD5 | gF2.gE3 | | gC1 |
| gA1.gA2 | gB5.gB4 | gC5.g- | gD6.gE5 | gF2.gF1 | | gC2 |
| gA1.gB1 | gB5.gB6 | gC6.gB5 | gD6.gE6 | gF3.gE2 | | gC3 |
| gA1.gB2 | gB5.gC4 | gC6.gB6 | gD6.g- | gF3.gE3 | | gC4 |
| gA2.gA1 | gB5.gC5 | gC6.gC5 | gE1.gD1 | gF3.gE4 | | gC5 |
| gA2.gB1 | gB5.gC6 | gC6.gD5 | gE1.gD2 | gF3.gF2 | | gC6 |
| gA2.gB2 | gB6.gA5 | gC6.gD6 | gE1.gE2 | gF4.gE3 | | gD1 |
| gA2.gB3 | gB6.gA6 | gC6.g- | gE1.gF1 | gF4.gE4 | | gD2 |
| gA3.gA2 | gB6.gB5 | gD1.gC1 | gE1.gF2 | gF4.gE5 | | gD3 |
| gA3.gB2 | gB6.gC5 | gD1.gC2 | gE2.gD1 | gF4.gF5 | | gD4 |
| gA3.gB3 | gB6.gC6 | gD1.gD2 | gE2.gD2 | gF5.gE4 | | gD5 (protected) |
| gA3.gB4 | gC1.gB1 | gD1.gE1 | gE2.gD3 | gF5.gE5 | | gD6 |
| gA4.gA5 | gC1.gB2 | gD1.gE2 | gE2.gE1 | gF5.gE6 | | gE1 |
| gA4.gB3 | gC1.gC2 | gD2.gC1 | gE2.gE3 | gF5.gF6 | | gE2 |
| gA4.gB4 | gC1.gD1 | gD2.gC2 | gE2.gF1 | gF6.gE5 | | gE3 |
| gA4.gB5 | gC1.gD2 | gD2.gC3 | gE2.gF2 | gF6.gE6 | | gE4 |
| gA5.gA6 | gC2.gB1 | gD2.gD1 | gE3.gD2 | gF6.gF5 | | gE5 |
| gA5.gB4 | gC2.gB2 | gD2.gD3 | gE3.gD3 | | | gE6 |
| gA5.gB5 | gC2.gB3 | gD2.gE1 | gE3.gD4 | | | gF1 |
| gA5.gB6 | gC2.gC1 | gD2.gE2 | gE3.gE2 | | | gF2 |
| gA6.gA5 | gC2.gC3 | gD2.gE3 | gE3.gE4 | | | gF3 (impassable) |
| gA6.gB5 | gC2.gD1 | gD3.gC2 | gE3.gF2 | | | gF4 (impassable) |
| gA6.gB6 | gC2.gD2 | gD3.gC3 | gE4.gD3 | | | gF5 |
| gB1.gA1 | gC2.gD3 | gD3.gC4 | gE4.gD4 | | | gF6 |
| gB1.gA2 | gC3.gB2 | gD3.gD2 | gE4.gD5 | | | $g^-$ |
| gB1.gB2 | gC3.gB3 | gD3.gD4 | gE4.gE3 | | | |
| gB1.gC1 | gC3.gB4 | gD3.gE2 | gE4.gE5 | | | |
| gB1.gC2 | gC3.gC2 | gD3.gE3 | gE4.gF5 | | | |
| gB2.gA1 | gC3.gC4 | gD3.gE4 | gE4.g- | | | |
| gB2.gA2 | gC3.gD2 | gD4.gC3 | gE5.gD4 | | | |
| gB2.gB1 | gC3.gD3 | gD4.gC4 | gE5.gD5 | | | |
| gB2.gB3 | gC3.gD4 | gD4.gC5 | gE5.gD6 | | | |
| gB2.gC1 | gC4.gB3 | gD4.gD3 | gE5.gE4 | | | |
| gB2.gC2 | gC4.gB4 | gD4.gD5 | gE5.gE6 | | | |
| gB2.gC3 | gC4.gB5 | gD4.gE3 | gE5.gF5 | | | |
| gB3.gA2 | gC4.gC3 | gD4.gE4 | gE5.gF6 | | | |
| gB3.gB2 | gC4.gC5 | gD4.gE5 | gE5.g- | | | |
| gB3.gB4 | gC4.gD3 | gD4.g- | gE6.gD5 | | | |
| gB3.gC2 | gC4.gD4 | gD5.gC4 | gE6.gD6 | | | |
| gB3.gC3 | gC4.gD5 | gD5.gC5 | gE6.gE5 | | | |
| gB3.gC4 | gC4.g- | gD5.gC6 | gE6.gF5 | | | |
| gB4.gA5 | gC5.gB4 | gD5.gD4 | gE6.gF6 | | | |
| gB4.gB3 | gC5.gB5 | gD5.gD6 | gE6.g- | | | |

***$G_{pm}$ (for all p in P)***

| | | | | | |
|---|---|---|---|---|---|
| p.m1.gA1 | p.m37.gB1 | p.m57.gE5 | p.m77.gE2 | p.m92.gC6 | p.m105.gB5 |
| p.m2.gA2 | p.m37.gC1 | p.m57.gF5 | p.m77.gE3 | p.m92.gD6 | p.m106.gB4 |
| p.m3.gA5 | p.m38.gB2 | p.m58.gE6 | p.m78.gE3 | p.m93.gC1 | p.m106.gB5 |
| p.m4.gA6 | p.m38.gC2 | p.m58.gF6 | p.m78.gE4 | p.m93.gD1 | p.m106.gB6 |
| p.m5.gB1 | p.m39.gB3 | p.m59.gA1 | p.m79.gE4 | p.m93.gE1 | p.m107.gC1 |
| p.m6.gB2 | p.m39.gC3 | p.m59.gA2 | p.m79.gE5 | p.m94.gC2 | p.m107.gC2 |
| p.m7.gB3 | p.m40.gB4 | p.m60.gA5 | p.m80.gE5 | p.m94.gD2 | p.m107.gC3 |
| p.m8.gB4 | p.m40.gC4 | p.m60.gA6 | p.m80.gE6 | p.m94.gE2 | p.m108.gC2 |
| p.m9.gB5 | p.m41.gB5 | p.m61.gB1 | p.m81.gF1 | p.m95.gC3 | p.m108.gC3 |
| p.m10.gB6 | p.m41.gC5 | p.m61.gB2 | p.m81.gF2 | p.m95.gD3 | p.m108.gC4 |
| p.m11.gC1 | p.m42.gB6 | p.m62.gB2 | p.m82.gF5 | p.m95.gE3 | p.m109.gC3 |
| p.m12.gC2 | p.m42.gC6 | p.m62.gB3 | p.m82.gF6 | p.m96.gC4 | p.m109.gC4 |
| p.m13.gC3 | p.m43.gC1 | p.m63.gB3 | p.m83.gA1 | p.m96.gD4 | p.m109.gC5 |
| p.m14.gC4 | p.m43.gD1 | p.m63.gB4 | p.m83.gB1 | p.m96.gE4 | p.m110.gC4 |
| p.m15.gC5 | p.m44.gC2 | p.m64.gB4 | p.m83.gC1 | p.m97.gC5 | p.m110.gC5 |
| p.m16.gC6 | p.m44.gD2 | p.m64.gB5 | p.m84.gA2 | p.m97.gD5 | p.m110.gC6 |
| p.m17.gD1 | p.m45.gC3 | p.m65.gB5 | p.m84.gB2 | p.m97.gE5 | p.m111.gD1 |
| p.m18.gD2 | p.m45.gD3 | p.m65.gB6 | p.m84.gC2 | p.m98.gC6 | p.m111.gD2 |
| p.m19.gD3 | p.m46.gC4 | p.m66.gC1 | p.m85.gA5 | p.m98.gD6 | p.m111.gD3 |
| p.m20.gD4 | p.m46.gD4 | p.m66.gC2 | p.m85.gB5 | p.m98.gE6 | p.m112.gD2 |
| p.m21.gD5 | p.m47.gC5 | p.m67.gC2 | p.m85.gC5 | p.m99.gD1 | p.m112.gD3 |
| p.m22.gD6 | p.m47.gD5 | p.m67.gC3 | p.m86.gA6 | p.m99.gE1 | p.m112.gD4 |
| p.m23.gE1 | p.m48.gC6 | p.m68.gC3 | p.m86.gB6 | p.m99.gF1 | p.m113.gD3 |
| p.m24.gE2 | p.m48.gD6 | p.m68.gC4 | p.m86.gC6 | p.m100.gD2 | p.m113.gD4 |
| p.m25.gE3 | p.m49.gD1 | p.m69.gC4 | p.m87.gB1 | p.m100.gE2 | p.m113.gD5 |
| p.m26.gE4 | p.m49.gE1 | p.m69.gC5 | p.m87.gC1 | p.m100.gF2 | p.m114.gD4 |
| p.m27.gE5 | p.m50.gD2 | p.m70.gC5 | p.m87.gD1 | p.m101.gD5 | p.m114.gD5 |
| p.m28.gE6 | p.m50.gE2 | p.m70.gC6 | p.m88.gB2 | p.m101.gE5 | p.m114.gD6 |
| p.m29.gF1 | p.m51.gD3 | p.m71.gD1 | p.m88.gC2 | p.m101.gF5 | p.m115.gE1 |
| p.m30.gF2 | p.m51.gE3 | p.m71.gD2 | p.m88.gD2 | p.m102.gD6 | p.m115.gE2 |
| p.m31.gF5 | p.m52.gD4 | p.m72.gD2 | p.m89.gB3 | p.m102.gE6 | p.m115.gE3 |
| p.m32.gF6 | p.m52.gE4 | p.m72.gD3 | p.m89.gC3 | p.m102.gF6 | p.m116.gE2 |
| p.m33.gA1 | p.m53.gD5 | p.m73.gD3 | p.m89.gD3 | p.m103.gB1 | p.m116.gE3 |
| p.m33.gB1 | p.m53.gE5 | p.m73.gD4 | p.m90.gB4 | p.m103.gB2 | p.m116.gE4 |
| p.m34.gA2 | p.m54.gD6 | p.m74.gD4 | p.m90.gC4 | p.m103.gB3 | p.m117.gE3 |
| p.m34.gB2 | p.m54.gE6 | p.m74.gD5 | p.m90.gD4 | p.m104.gB2 | p.m117.gE4 |
| p.m35.gA5 | p.m55.gE1 | p.m75.gD5 | p.m91.gB5 | p.m104.gB3 | p.m117.gE5 |
| p.m35.gB5 | p.m55.gF1 | p.m75.gD6 | p.m91.gC5 | p.m104.gB4 | p.m118.gE4 |
| p.m36.gA6 | p.m56.gE2 | p.m76.gE1 | p.m91.gD5 | p.m105.gB3 | p.m118.gE5 |
| p.m36.gB6 | p.m56.gF2 | p.m76.gE2 | p.m92.gB6 | p.m105.gB4 | p.m118.gE6 |

***G*** $_{pm}$ ***(for all p in P) (continued)***

| | | | | | |
|---|---|---|---|---|---|
| p.m119.gA1 | p.m132.gD3 | p.m145.gC5 | p.m159.gB1 | p.m172.gD2 | p.m185.gC6 |
| p.m119.gA2 | p.m132.gE2 | p.m146.gB5 | p.m159.gA2 | p.m172.gE2 | p.m186.gC1 |
| p.m119.gB1 | p.m133.gD3 | p.m146.gC5 | p.m159.gB2 | p.m173.gE2 | p.m186.gC2 |
| p.m120.gA5 | p.m133.gD4 | p.m146.gC6 | p.m160.gB4 | p.m173.gD3 | p.m186.gD2 |
| p.m120.gA6 | p.m133.gE3 | p.m147.gC1 | p.m160.gA5 | p.m173.gE3 | p.m187.gC2 |
| p.m120.gB5 | p.m134.gD4 | p.m147.gD1 | p.m160.gB5 | p.m174.gE3 | p.m187.gC3 |
| p.m121.gB1 | p.m134.gD5 | p.m147.gD2 | p.m161.gB5 | p.m174.gD4 | p.m187.gD3 |
| p.m121.gB2 | p.m134.gE4 | p.m148.gC2 | p.m161.gA6 | p.m174.gE4 | p.m188.gC3 |
| p.m121.gC1 | p.m135.gD5 | p.m148.gD2 | p.m161.gB6 | p.m175.gE4 | p.m188.gC4 |
| p.m122.gB2 | p.m135.gD6 | p.m148.gD3 | p.m162.gC1 | p.m175.gD5 | p.m188.gD4 |
| p.m122.gB3 | p.m135.gE5 | p.m149.gC3 | p.m162.gB2 | p.m175.gE5 | p.m189.gC4 |
| p.m122.gC2 | p.m136.gE1 | p.m149.gD3 | p.m162.gC2 | p.m176.gE5 | p.m189.gC5 |
| p.m123.gB3 | p.m136.gE2 | p.m149.gD4 | p.m163.gC2 | p.m176.gD6 | p.m189.gD5 |
| p.m123.gB4 | p.m136.gF1 | p.m150.gC4 | p.m163.gB3 | p.m176.gE6 | p.m190.gC5 |
| p.m123.gC3 | p.m137.gE2 | p.m150.gD4 | p.m163.gC3 | p.m177.gF1 | p.m190.gC6 |
| p.m124.gB4 | p.m137.gE3 | p.m150.gD5 | p.m164.gC3 | p.m177.gE2 | p.m190.gD6 |
| p.m124.gB5 | p.m137.gF2 | p.m151.gC5 | p.m164.gB4 | p.m177.gF2 | p.m191.gD1 |
| p.m124.gC4 | p.m138.gE5 | p.m151.gD5 | p.m164.gC4 | p.m178.gF5 | p.m191.gD2 |
| p.m125.gB5 | p.m138.gE6 | p.m151.gD6 | p.m165.gC4 | p.m178.gE6 | p.m191.gE2 |
| p.m125.gB6 | p.m138.gF5 | p.m152.gD1 | p.m165.gB5 | p.m178.gF6 | p.m192.gD2 |
| p.m125.gC5 | p.m139.gA1 | p.m152.gE1 | p.m165.gC5 | p.m179.gA1 | p.m192.gD3 |
| p.m126.gC1 | p.m139.gB1 | p.m152.gE2 | p.m166.gC5 | p.m179.gA2 | p.m192.gE3 |
| p.m126.gC2 | p.m139.gB2 | p.m153.gD2 | p.m166.gB6 | p.m179.gB2 | p.m193.gD3 |
| p.m126.gD1 | p.m140.gA2 | p.m153.gE2 | p.m166.gC6 | p.m180.gA5 | p.m193.gD4 |
| p.m127.gC2 | p.m140.gB2 | p.m153.gE3 | p.m167.gD1 | p.m180.gA6 | p.m193.gE4 |
| p.m127.gC3 | p.m140.gB3 | p.m154.gD3 | p.m167.gC2 | p.m180.gB6 | p.m194.gD4 |
| p.m127.gD2 | p.m141.gA5 | p.m154.gE3 | p.m167.gD2 | p.m181.gB1 | p.m194.gD5 |
| p.m128.gC3 | p.m141.gB5 | p.m154.gE4 | p.m168.gD2 | p.m181.gB2 | p.m194.gE5 |
| p.m128.gC4 | p.m141.gB6 | p.m155.gD4 | p.m168.gC3 | p.m181.gC2 | p.m195.gD5 |
| p.m128.gD3 | p.m142.gB1 | p.m155.gE4 | p.m168.gD3 | p.m182.gB2 | p.m195.gD6 |
| p.m129.gC4 | p.m142.gC1 | p.m155.gE5 | p.m169.gD3 | p.m182.gB3 | p.m195.gE6 |
| p.m129.gC5 | p.m142.gC2 | p.m156.gD5 | p.m169.gC4 | p.m182.gC3 | p.m196.gE1 |
| p.m129.gD4 | p.m143.gB2 | p.m156.gE5 | p.m169.gD4 | p.m183.gB3 | p.m196.gE2 |
| p.m130.gC5 | p.m143.gC2 | p.m156.gE6 | p.m170.gD4 | p.m183.gB4 | p.m196.gF2 |
| p.m130.gC6 | p.m143.gC3 | p.m157.gE1 | p.m170.gC5 | p.m183.gC4 | p.m197.gE4 |
| p.m130.gD5 | p.m144.gB3 | p.m157.gF1 | p.m170.gD5 | p.m184.gB4 | p.m197.gE5 |
| p.m131.gD1 | p.m144.gC3 | p.m157.gF2 | p.m171.gD5 | p.m184.gB5 | p.m197.gF5 |
| p.m131.gD2 | p.m144.gC4 | p.m158.gE5 | p.m171.gC6 | p.m184.gC5 | p.m198.gE5 |
| p.m131.gE1 | p.m145.gB4 | p.m158.gF5 | p.m171.gD6 | p.m185.gB5 | p.m198.gE6 |
| p.m132.gD2 | p.m145.gC4 | p.m158.gF6 | p.m172.gE1 | p.m185.gB6 | p.m198.gF6 |

# Data

***dist***$_q$

| | |
|---|---|
| gA1 | 25 |
| gA2 | 20 |
| gA3 | 15 |
| gA4 | 15 |
| gA5 | 15 |
| gA6 | 15 |
| gB1 | 20 |
| gB2 | 15 |
| gB3 | 10 |
| gB4 | 10 |
| gB5 | 10 |
| gB6 | 10 |
| gC1 | 20 |
| gC2 | 15 |
| gC3 | 10 |
| gC4 | 5 |
| gC5 | 5 |
| gC6 | 5 |
| gD1 | 20 |
| gD2 | 15 |
| gD3 | 10 |
| gD4 | 5 |
| gD5 | 0 |
| gD6 | 5 |
| gE1 | 20 |
| gE2 | 15 |
| gE3 | 10 |
| gE4 | 5 |
| gE5 | 5 |
| gE6 | 5 |
| gF1 | 20 |
| gF2 | 15 |
| gF3 | 10 |
| gF4 | 10 |
| gF5 | 10 |
| gF6 | 10 |

***teth_range***$_{p''}$

| | |
|---|---|
| Helo1 | 10.0 |

***trans***$_p$

| | |
|---|---|
| Surf1 | 0.5 |
| Helo1 | 0.1 |
| SSN1 | 0.7 |

***r***$_{psq}$

| | Surf1.active | Helo1.active | SSN1.passive | SSN1.active |
|---|---|---|---|---|
| gA1 | 0.42 | 0.5 | 0.45 | 0.63 |
| gA2 | 0.42 | 0.5 | 0.45 | 0.63 |
| gA3 | 0.56 | 0.67 | 0.6 | 0.84 |
| gA4 | 0.7 | 0.84 | 0.75 | 1.05 |
| gA5 | 0.91 | 1.09 | 0.97 | 1.36 |
| gA6 | 0.95 | 1.14 | 1.02 | 1.43 |
| gB1 | 0.42 | 0.5 | 0.45 | 0.63 |
| gB2 | 0.49 | 0.59 | 0.53 | 0.74 |
| gB3 | 0.56 | 0.67 | 0.6 | 0.84 |
| gB4 | 0.7 | 0.84 | 0.75 | 1.05 |
| gB5 | 0.88 | 1.05 | 0.94 | 1.32 |
| gB6 | 1.03 | 1.24 | 1.11 | 1.55 |
| gC1 | 0.56 | 0.67 | 0.6 | 0.84 |
| gC2 | 0.63 | 0.76 | 0.68 | 0.95 |
| gC3 | 0.7 | 0.84 | 0.75 | 1.05 |
| gC4 | 0.84 | 1.01 | 0.9 | 1.26 |
| gC5 | 0.91 | 1.1 | 0.98 | 1.37 |
| gC6 | 0.97 | 1.16 | 1.03 | 1.45 |
| gD1 | 0.91 | 1.09 | 0.98 | 1.37 |
| gD2 | 1.01 | 1.22 | 1.08 | 1.52 |
| gD3 | 0.77 | 0.92 | 0.83 | 1.16 |
| gD4 | 0.63 | 0.76 | 0.68 | 0.95 |
| gD5 | 0.7 | 0.84 | 0.75 | 1.05 |
| gD6 | 0.7 | 0.84 | 0.75 | 1.05 |
| gE1 | 0.99 | 1.19 | 1.06 | 1.48 |
| gE2 | 1.03 | 1.24 | 1.1 | 1.55 |
| gE3 | 0.63 | 0.76 | 0.68 | 0.95 |
| gE4 | 0.49 | 0.59 | 0.53 | 0.74 |
| gE5 | 0.56 | 0.67 | 0.6 | 0.84 |
| gE6 | 0.56 | 0.67 | 0.6 | 0.84 |
| gF1 | 0.95 | 1.14 | 1.02 | 1.43 |
| gF2 | 0.88 | 1.05 | 0.94 | 1.31 |
| gF3 | 0.56 | 0.67 | 0.6 | 0.84 |
| gF4 | 0.42 | 0.5 | 0.45 | 0.63 |
| gF5 | 0.42 | 0.5 | 0.45 | 0.63 |
| gF6 | 0.42 | 0.5 | 0.45 | 0.63 |

These data are identical to Example One:

***range***$_{m'm''}$

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